Impact a circular cylinder with a flat on an elastic layer

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Abstract. In the work the comparison of the results of solving two plane problems is performed: the impact of a circular cylinder with a plane platform parallel to the cylinder axle (the flat) with an elastic layer and a second − plane strain state of nonstationary interaction of a circular cylinder with a flat with an elastic layer in a purely elastic and elastic-plastic mathematical formulation corresponding. The first contact occurs along the plane of the flat. A good coincidence of the results of the second problem at an elastic stage with the results of the first problem is shown. In the author's works a new approach was developed to solve plane and tree dimension problems of impact and non-stationary interaction in an elastoplastic formulation. The crack growing was simulated using an elastoplastic mathematical model. The numerical solution was obtained using the finite difference method scheme.

The use of an elastic-plastic formulation makes it possible: 1) determine the stress-strain state at the points determined by the partitioning grid of the computational domain, not only on the surface; 2) to give a reliable description of the development of plastic deformations − the stage corresponding to plasticity is a continuation of the elastic stage; 3) reliably determine the destruction toughness. A method has been developed for calculating plastic strain fields and destruction toughness of the material using the solutions of dynamic plane problems of the stress-strain state in an elastoplastic formulation taking into account possible material unloading; 4) to verify and calibrate the solution of problems in an elastoplastic formulation for the first steps by time when the deformation process is elastic, it is convenient to use the solution of the corresponding elastic problem.

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INTRODUCTION

The approach [3 − 7] for solving the dynamic problems, developed by V.D. Kubenko makes it possible to determine the stress-strain state only on the surface of the medium into which the drummer penetrates. In addition, this approach does not allow to investigate the impact of elastic shells of S.P. Tymoshenko type. To the equations describing the dynamics of the shell, the Laplace transform and the development to Fourier trigonometric series are applied. After returning to the space of the originals and using theorem on convolution in integral expressions, the components of the series of normal and tangential displacements of the median surface of the shells of the S.P. Tymoshenko type some nuclei will have asymptotic $O(1)$. Therefore, with increasing order of a reduced system of integral equations of Volterra of the second kind [1 − 3], the determinant of the system of linear algebraic equations will be indefinitely enlarged – it will seem that the matrix of this system is weak conditioned. However, if use shell of Kirchhoff − Love type [4 − 7], then when solving problems of impact, the convergence of the solution will be guaranteed. This led [8 − 12] to the expediency of developing other mathematical approaches and models. In [13 − 17], a new approach to solving problems of impact and non-stationary interactions in an elastic-plastic mathematical setting was developed [18 − 21].

In this paper it was compared the results of solving two plane problems of the motion of a circular rigid cylinder with a flat on an elastic layer: 1) impact within a strictly elastic model; 2) no stationary interaction in elastoplastic formulation. At the initial moment the circular cylinder contacted with the surface of the layer along the plane of the flat.

PROBLEM FORMULATION

Firstproblem.Thehardcircularcylinderwitht heflatmovesverticallydownperpendiculartothes urfaceoftheelasticlayer $0 \le z \le H$ and contacts it along the lane $\{|x| \le d;\ z = 0\}$, where d – half width of the flat. As in [1 − 3] we associate a cylindrical coordinate system rOz' with a moving cylinder, axis z coincides with the axle of the cylinder. We associate with a layer a fixed Cartesian coordinate system xyz [5 − 7].

The stamp penetrates (Fig.1) an elastic layer at a speed $V_T(t)$, $(0 \le t \le T)$ with initial value $V_0 = V_T(0)$, where T is the time of interaction of a stamp with a layer. We introduce dimensionless variables.

$$t' = \frac{C_0 t}{R},\ x' = \frac{x}{R},\ z' = \frac{z}{R},\ u_t' = \frac{u_t}{R},$$

$$\sigma_{ij}' = \frac{\sigma_{ij}}{K},\ v_T' = \frac{v_T}{C_0},\ w_T' = \frac{w_T}{R},\ p' = \frac{p}{KR},$$

$$q' = \frac{q}{KR},\ M' = \frac{M}{\rho R^2},\ (i, j = x, y, z),$$

$$\beta^2 = \frac{C_S^2}{C_0^2} = \frac{\mu}{K},\ \alpha^2 = \frac{C_p^2}{C_0^2} = \left(1 + \frac{4\mu}{3K}\right),$$

$$C_0^2 = \frac{K}{\rho},\ b^2 = \frac{\beta^2}{\alpha^2} = \frac{3\mu}{3K + 4\mu}.$$

where ρ, μ, K, C_p and C_s is the density, the displacement module, the volume deformation module and the velocity of the waves in the elastic layer.

The motion [1 − 3] of an elastic layer is described by scalar potentials φandψ, which satisfy the wave equations [5 − 7]:

$$\Delta\varphi = \frac{\partial^2 \varphi}{\alpha^2 \partial t^2},\ \Delta\psi = \frac{\partial^2 \psi}{\beta^2 \partial t^2},\ \Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

When problem solving an approach is used [1 − 3], which makes it possible at the initial stage of penetration to identify the linear coordinates along the surface of the layer and body [5 − 7]. As a result, approximate ratios will be executed.

$$r \approx \theta,\ \mathrm{ctg}\theta \approx 1/\theta. \tag{1}$$

In the contact area, taking into account (1), there is a relationship between u_z and pressure p.

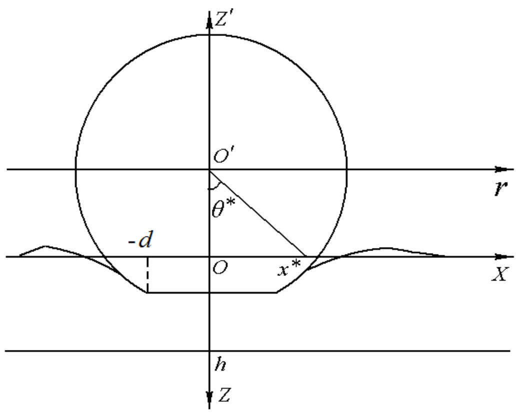

Fig.1. Scheme of the system stamp-layer

$$u_z(t, x, 0) = w_T(t) - H(|x| - d) \times$$

$$\times\left(1-\sqrt{1-(|x|-d)^2}\right), \quad (2)$$

$$w_T(t)=\int_0^t V(\tau)d\tau,\ p(t,x)=-\sigma_{zz}(t,x,0),\ |x|<x^*.$$

Linear zed boundary conditions are as follows:

$$\left.\frac{\partial u_z}{\partial t}\right|_{z=0}\equiv V(t,x)=v_T(t),\quad |x|<x^*(t), \quad (3)$$

$$\sigma_{zz}|_{z=0}=0,\ |x|>x^*(t), \quad (4)$$
$$\sigma_{zx}|_{z=0}=0,\ |x|<\infty.$$

On the surface of the layer $z=h$ there are conditions of hard jamming.

For interaction time $0\le t\le T$ select a rectangle $\{|x|\le l,\ 0\le z\le h\}$, which is occupied by the medium, and the task of impact on the layer can be considered as a problem of impact on a rectangle. The width of the rectangle l is chosen so that the perturbation waves do not reach its boundaries:

$$|x|=l\left(l>\alpha(T-t_0)+x^*(t_0),\ \left.\frac{dx^*}{dt}\right|_{t=t_0}\right).$$

For certainty, we choose the condition of a smooth sliding contact on the lateral surface of the rectangle. Initial conditions of the problem are zero.

$$u_x|_{|x|=l}=0,\ \ \sigma_{zx}|_{|x|=l}=0, \quad (5)$$
$$\varphi|_{t=0}=\left.\frac{\partial\varphi}{\partial t}\right|_{t=0}=0,\ \psi|_{t=0}=\left.\frac{\partial\psi}{\partial t}\right|_{t=0}=0.$$

The motion of a cylinder as a body outlines the second law of Newton

$$M\frac{\partial^2 w_T}{\partial t^2}=-F(t),\ V_T(0)=V_0,\ w_T(0)=0, \quad (6)$$

where $F(t)$ – the reaction strength of an elastic layer, which is determined considering (2), (4) as an integral from pressure in the contact region:

$$F(t)=2\int_0^{x^*(t)}p(t,x)dx.$$

The boundary of the contact area x^*, taking into account the motion of the particles of the medium and retarding the penetration of the cylinder in the elastic medium, is determined from the condition:

$$w_T(t)-u_z(t,x^*,0)-H(|x^*|-d)\times$$
$$\times\left(1-\sqrt{1-(|x^*|-d)^2}\right)=\begin{cases}0, & |x|\le x^*(t)\\ \varepsilon<0, & |x|>x^*(t).\end{cases}$$

Second problem. Its mathematical formulation is the same as in [10, 11, 13, 16, 17]. Deformation of a beam sample in the form of a rectangle $\Sigma=L\times h$ $(-L/2\le x\le L/2;0\le y\le h)$ is considered. The beam samples based on a completely rigid basis along $\{-L/2\le x\le L/2;\ \ y=0\}$. The thickness of the sample is considered so large that it would be possible to use the dependences of the plane strain state.

On top of the body a completely hard impact or contacts with beam along the segment $\{|x|\le d;\ \ y=h\}$. Its effect on the body in the contact area will be replaced evenly distributed normal stress $-P$ which varies with time as a linear function $P=p_{01}+p_{02}\times(t/\Delta t-1)$, where Δt – increment of time.

Given the symmetry of the deformation process with respect to the line $x=0$, only the right part of the transverse section of the body is considered further (Fig.2).

As a result of the impact load we will consider that the material is elastic-plastic with strengthening and calculation of fields of stresses, deformations and their increments, in particular, increments of plastic deformation intensity $d\varepsilon_i^p$ and the parameter of the Odquist $\kappa = \int d\varepsilon_i^p$ will be carried out on the basis of numerical solution of the corresponding dynamic elastic-plastic problem.

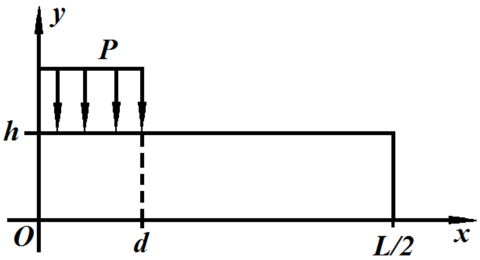

Fig.2. Scheme of the system stamp-layer. Second problem

When calculating the dynamic fields of stresses and deformations the interaction of wave fields, reflection from the boundary of the body were not taken into account.

The equations of motion for a plane problem are used

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u_x}{\partial t^2}, \qquad (7)$$
$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 u_y}{\partial t^2},$$

where ρ -material density.

The boundary conditions of the problem, which follow from the assumption that the region of application of the forces of the reaction of the supports is unaltered, as well as the determination of the supporting reactions have done using static methods, are written:

$$x = 0; \;\; 0 < y < B: \;\; u_x = 0, \;\; \sigma_{xy} = 0;$$
$$x = \frac{L}{2}, \;\; 0 < y < B: \;\; \sigma_{xx} = 0, \;\; \sigma_{xy} = 0;$$
$$y = 0, \;\; 0 < x < \frac{L}{2}: \;\; u_y = 0, \;\; \sigma_{xy} = 0; \qquad (8)$$
$$y = h, \;\; 0 < x < d: \;\; \sigma_{yy} = -P, \;\; \sigma_{xy} = 0;$$
$$y = h, \;\; d < x < \frac{L}{2}: \;\; \sigma_{yy} = 0, \;\; \sigma_{xy} = 0.$$

Initial conditions are zero. In the basis of the defining relations of the mechanical model, the theory of no isothermal plastic flow of the medium with the strengthening under the condition of Huber-Mises fluidity [10, 11, 13, 16, 17] was applied. The effects of creep and temperature expansion are neglected. Then, considering the components of the deformation tensor by the sum of the elastic and plastic components of it [10, 11, 13, 16, 17], we obtain for them

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p, \;\; \varepsilon_{ij}^e = \frac{1}{2G} s_{ij} + K\sigma + \varphi_1,$$
$$d\varepsilon_{ij}^p = s_{ij} d\lambda, \qquad (9)$$

here $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma$ – components of the stress tensor deviator; δ_{ij} - a symbol of Kronecker; G – displacement module; $K_1 = (1-2\nu)/(3E)$; E – modulus of elasticity; ν -Poisson's coefficient; $K = 3K_1$ – the volume compression module, which binds to the ratio $\varepsilon = K\sigma + \varphi_1$ volumetric expansion 3ε (temperature expansion $\varphi_1 \equiv 0$); $\sigma = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$-average tension; $d\lambda$ – some scalar function, which is determined by the condition of plasticity (the shape of the surface of the load) and in view of the above, its choice quadratic ally depends on the components of the deviant stress s_{ij} [10, 11, 13, 16, 17]. The material is strengthened with a strengthening factor η_* [10, 11, 13, 16, 17]:

$$\sigma_S(T) = \sigma_{02}(T_0)\left(1 + \frac{\kappa(T)}{\varepsilon_0}\right)^{\eta_*},$$
$$\varepsilon_0 = \frac{\sigma_{02}(T_0)}{E}, \; T_0 = 20^\circ\mathrm{C}, \qquad (10)$$

where $\sigma_S(T)$ - the line of fluidity after strengthening the material at a temperature T.

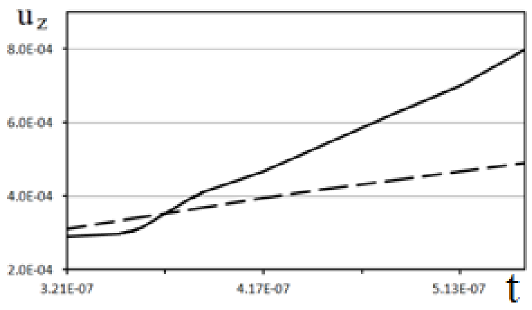

Fig. 3. The vector of the displacement component u_z

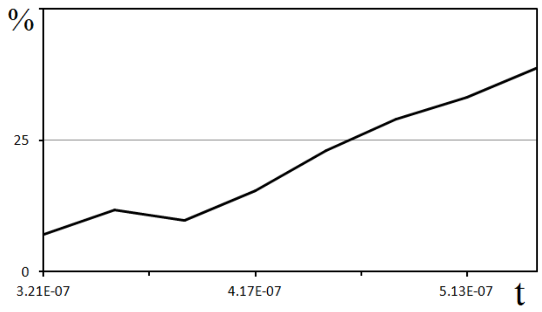

Fig. 4. Rejection of results

SCHEME AND METHODS NUMERIC REALIZATION

The scheme, methods of solving and numerical realization of the first problem are the same as in [1, 2] and for the second problem – are the same as in [10, 11, 13, 16, 17]. The application of the finite difference method to the solution of wave equations is justified in [22] and ensure the accuracy of calculations with an error of not more than $O\left((\Delta x)^2 + (\Delta y)^2 + (\Delta t)^2\right)$.

As example an aluminum layer was chosen $\mu = 0.3582K$. The figures below represent the results corresponding to the calculation when $V_0 = 0.0002$, $h/R = 0.01$, $M = 0.001$, $l = 0.6$, $T = 0.05$, $h = 0.4$, $d = 0.02$; Δt =4.166667E–5.

The results of the calculation for the second problem are obtained for the following parameters values: the coefficient of strengthening the material η_* =0,05; L=600 mm; h=400 mm; d =2 mm; p_{01} = 10.1 MPa; p_{02} =4,04 MPa; M=80; N=101. The smallest step of the partition was near the upper surface and equaled 0,01 mm, (Δx_{min} = 0,01 mm; Δy_{min} = 0,01 mm (only the first three layers)), T =50°.

In the elastic-plastic model, the axle Oy coincides with the axis Oz'. In Fig.3 shows the components of the displacement vectors u_z in the first problem and $-u_y$ in the second are shown at the Fig.3 and denoted by u_z. The components of vector displacements u_z in the point (0, 0) in the center of the contact area for the first problem in the elastic model (dashed line) and at the point (0.01, 399.99) for the second problem in the elastic-plastic formulation (solid line) were compared.

The percentage of rejection of the displacement values u_z received for the first and second tasks is shown at the Fig.4. A period has been found for which this deviation does not exceed 8%.

CONCLUSIONS

The results of solving plane problems of the impact of a circular cylinder with a flat in an elastic model and a non-stationary interaction in an elastic-plastic mathematical setting at the elastic stage coincide well. The use of elastoplastic formulation makes it possible:

1. Determine the stress-strain state at the points determined by the grid of the breakdown of the calculated region, and not only on the surface.
2. Give a plausible description of the development of plastic deformations. The step corresponding to plasticity is the continuation of the elastic stage.
3. Authentically determine the destruction toughness K_{Ic}.
4. To verify the solving of problems in the elastic-plastic formulation of the first steps in time, when the deformation process is elastic, it is convenient to use the solution of the corresponding elastic problem.

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Удар кругового цилиндра с лыской по упругому слою

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Аннотация. Задачи ударного нагружения твердых деформируемых тел остаются актуальными и исследуются в самых разных постановках. В работе проводиться сравнение результатов решения двух плоских задач: удара кругового цилиндра с плоской площадкой параллельной оси цилиндра (лыской) с упругим слоем и второй − о плоском деформированном состоянии при нестационарном взаимодействии кругового цилиндра с лыской с упругим слоем в чисто упругой и упругопластической математических постановках соответственно. Первоначальный контакт происходит по плоскости лыски. Показано хорошее совпадение результатов второй задачи на начальном упругом этапе с результатами первой задачи.

Разработан новый подход решения задач удара и нестационарного взаимодействия в упругопластической постановке. Численное решение получено с использованием схемы метода конечных разностей. Использование упругопластической постановки дает возможность: 1) определить напряженно-деформированное состояние в точках, определяемых сеткой разбиения расчетной области, а не только на поверхности; 2) дать достоверное описание развития пластических деформаций − этап, отвечающий пластичности, является продолжением упругого этапа; 3) достоверно определить вязкость разрушения. Разработана методика расчета полей пластических деформаций и вязкости разрушения материала с использованием решения динамических плоских задач о напряженно деформированном состоянии в упругопластической постановке с учетом возможной разгрузки материала. 4. Для верификации решения задач в упруго-пластической постановке для первых шагов по времени, когда процесс деформации является упругим, удобно использовать решение соответствующей упругой задачи.

Ключевые слова: удар, упругий, упруго-пластический, слой, плоская задача, жесткий цилиндр.