Section 3. Information Technology

About one approach to the problems of impact of fine shells of the S.P. Timoshenko type

Vladislav Bogdanov

Progressive Research Solutions Pty. Ltd.
Buller Rd 28/2, Artarmon, Sydney, Australia, 2064
vladislav_bogdanov@hotmail.com, https://orcid.org/0000-0002-3424-1801

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ABSTRACT

The method of the outcoming dynamics problems to solve an infinite system of integral equations Volterra of the second kind and the convergence of this solution are well studied. Such approach has been successfully used for cases of the investigation of problems of the impact a hard bodies and an elastic fine shells of the Kirchhoff–Love type on elastic a half-space and a layer. In this paper an attempt is made to solve the plane and the axisymmetric problems of the impact of an elastic fine cylindric and spheric shells of the S.P. Timoshenko type on an elastic half-space using the method of the outcoming dynamics problems to solve an infinite system of integral equations Volterra of the second kind. The discretization using the Gregory methods for numerical integration and Adams for solving the Cauchy problem of the reduced infinite system of Volterra equations of the second kind results in a poorly defined system of linear algebraic equations: as the size of reduction increases the determinant of such a system to aim at infinity. This technique does not allow to solve plane and axisymmetric problems of dynamics for fine shells of the S.P. Timoshenko type and elastic bodies. It is shown that this approach is not acceptable for investigated in this paper the plane and the axisymmetric problems. This shows the limitations of this approach and leads to the feasibility of developing other mathematical approaches and models. It should be noted that to calibrate the computational process of deformation in the elasto-plastic formulation at the elastic stage, it is convenient and expedient to use the technique of the outcoming dynamics problems to solve an infinite system of integral equations Volterra of the second kind.

INTRODUCTION

The approach [2–6] for solving problems of dynamics, developed [7–9, 11] by V.D. Kubenko, makes it possible to determine the stress-strain state of elastic half-space and a layer during penetration of absolutely rigid bodies [2, 3, 8, 9, 11] and the stress-strain state of elastic Kirchhoff–Love type fine shells and elastic half-spaces and layers at their collision [4−7]. This led to the feasibility of developing other mathematical approaches and models. In [10, 12–15], a new approach to solving the problems of impact and nonstationary interaction in the elastoplastic mathematical formulation [16–20] was developed. In non-stationary problems, the action of the striker is replaced by a distributed load in the contact area, which changes according to a linear law [21–23]. The contact area remains constant. The developed elastoplastic formulation makes it possible to solve impact problems when the dynamic change in the boundary of the contact area is considered and based on this the movement of the striker as a solid body with a change in the penetration speed is taken into account. Also, such an elastoplastic formulation makes it possible to consider the hardening of the material in the process of nonstationary and impact interaction.

The solution of problems for elastic shells [24–27], elastic half-space [28–30], elastic layer [31], elastic rod [32, 33] were developed using method of the influence functions [34]. In [24] the process of non-stationary interaction of an elastic cylindrical shell with an elastic half-space at the so-called "supersonic" stage of interaction is studied. It is characterized by an excess of the expansion rate areas of contact interaction speed of propagation tension-compression waves in elastic half-space. The solution was developed using influence functions corresponding concentrated force or kinematic actions for an elastic isotropic half-space which were found and investigated in [34].

In this paper, we investigate the approach [4–7] for solving the axisymmetric problem of the impact of a spherical fine shell of the S.P. Timoshenko type on an elastic half-space.

It is shown that the approach [2–5], after the reduction of the infinite system of Volterra integral equations of the second kind [6 – 8, 11] and discretization using the Gregory methods for numerical integration and Adams for solving the Cauchy problem, a poorly defined system of linear algebraic equations is obtained for which the determinant of the matrix of coefficients increases indefinitely with increasing size of reduction.

PROBLEM FORMULATION

A thin elastic cylindrical shell comes into collision with the elastic half-space $z \geq 0$ with its lateral surface along the generatrix of the cylinder at the moment of time $t = 0$. We associate with the shell, as can be seen in Figure 1, a movable cylindrical coordinate system $r\theta z'$: θ – the polar angle, which is plotted from the positive direction of the *oz* axis, the *oy* axis coincides with the cylinder axis. Let us denote by $u_0(t,\theta)$, $w_0(t,\theta)$, $p(t,\theta)$, $q(t,\theta)$ the tangential and normal displacements of the points of the middle surface of the shell and the radial and tangential components of the distributed external load, which acts on the shell. We associate a fixed Cartesian coordinate system *xyz* with the half-space, so that the *Oz* axis is directed deep into the medium, the *Ox* axis is directed along the surface of the half-space, and the *Oy* axis is parallel to the generatrix of the cylinder. The shell thickness *h* is much less than the radius *R* of the middle surface of the shell ($h / R \leq 0,05$).

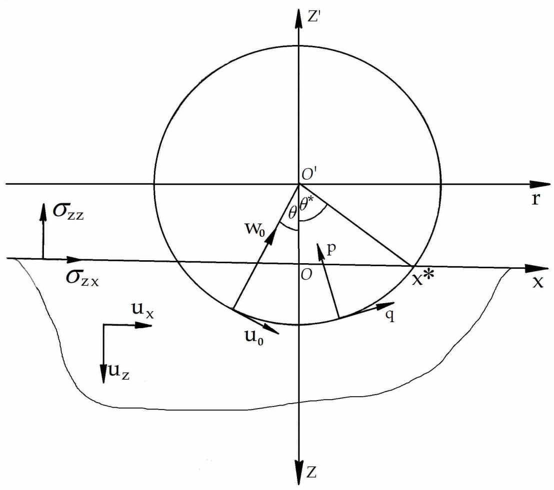

Fig. 1. Scheme of the system cylindrical shell – half space

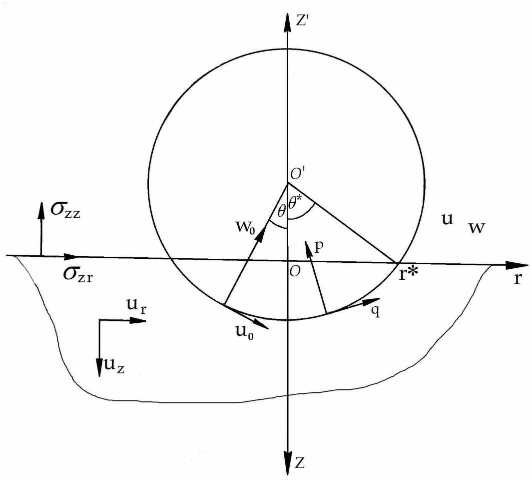

Fig. 2. Scheme of the system spherical shell – half space

In case of axisymmetric problem, a thin elastic spherical shell, moving perpendicular to the surface of the elastic half-space $z \geq 0$, reaches this surface at time *t*=0. We associate with the shell, as shown in Figure 2, a movable spherical coordinate system $r'\varphi'\theta$, where φ' – is the longitude of the radius vector *r*, θ – is the polar angle.

With the half-space we associate a fixed cylindrical coordinate system $r\varphi z$, the *Oz* axis is directed deep into the medium, φ – is the polar angle. Angle θ is plotted from the positive direction of the Oz axis.

The cylindric or spheric shell penetrates into the elastic medium at a speed $v_T(t)$, $(0 \le t \le T)$, the initial penetration rate is $V_0 = v_T(0)$, T – the time during which the shell interacts with the half-space. The shell thickness h is much less than the radius R of the middle surface of the shell ($h/R \le 0{,}05$).

Let us denote by $u_0(t,\theta)$, $w_0(t,\theta)$, $p(t,\theta)$, $q(t,\theta)$ the tangential and normal displacements of the points of the middle surface of the shell and the radial and tangential components of the distributed external load, which acts on the shell. With the half-space we associate a fixed cylindrical coordinate system $r\varphi z$, the Oz axis is directed deep into the medium, φ – is the polar angle. Angle θ is plotted from the positive direction of the Oz axis. The physical properties of the half-space material are characterized by elastic constants: volumetric expansion module K, shear modulus μ and density ρ.

Let's introduce dimensionless variables:

$$t' = \frac{C_0 t}{R},\quad \left\|\begin{matrix} x' \\ r' \end{matrix}\right\| = \frac{1}{R}\left\|\begin{matrix} x \\ r \end{matrix}\right\|,\quad z' = \frac{z}{R},\quad u_i' = \frac{u_i}{R},$$

$$u_0' = \frac{u_0}{R},\quad w_0' = \frac{w_0}{R},\quad \sigma_{ij}' = \frac{\sigma_{ij}}{K},\quad v_T' = \frac{v_T}{C_0}, \qquad (1)$$

$$w_T' = \frac{w_T}{R},\quad p' = \frac{p}{KR}\left\|\begin{matrix} 1 \\ 1/R \end{matrix}\right\|,\quad q' = \frac{q}{KR}\left\|\begin{matrix} 1 \\ 1/R \end{matrix}\right\|,$$

$$M' = \frac{M}{\rho R^2}\left\|\begin{matrix} 1 \\ 1/R \end{matrix}\right\|.\quad \left(i,j = \left\|\begin{matrix} x \\ r \end{matrix}\right\|, \left\|\begin{matrix} y \\ \varphi \end{matrix}\right\|, z\right)$$

$$\beta^2 = \frac{C_S^2}{C_0^2} = \frac{\mu}{K},\quad \alpha^2 = \frac{C_p^2}{C_0^2} = \left(1 + \frac{4\mu}{3K}\right),$$

$$C_0^2 = \frac{K}{\rho},\quad b^2 = \frac{\beta^2}{\alpha^2} = \frac{3\mu}{3K + 4\mu}.$$

here $\mathbf{u} = \left(u_{\left\|\begin{smallmatrix} x \\ r \end{smallmatrix}\right\|}, u_{\left\|\begin{smallmatrix} y \\ \varphi \end{smallmatrix}\right\|}, u_z\right)$ – is the vector of movement of points of the environment; σ_{zz}, $\sigma_{\left\|\begin{smallmatrix} xz \\ rz \end{smallmatrix}\right\|}$ – nonzero components of the stress tensor of the medium; M – is the shell running mass; $v_T(t)$, $w_T(t)$ – speed and movement of the shell as a solid. In what follows, we will use only dimensionless quantities, so we omit the dash. The elastic half-space and the spheric shell are in a state of axisymmetric deformation.

Differential equations (of the S.P. Timoshenko type) describing the dynamics of cylindrical (2) and spherical (3) shells and considering the shear and inertia of rotation of the transverse section, due to (1), take the following form [35, pp. 297, 307]:

$$\gamma_0^2 \frac{\partial^2 u_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial \theta^2} + (1 + a_4)\frac{\partial w_0}{\partial \theta} + a_4\Phi -$$
$$-a_4 u_0 + \beta_3 q,$$
$$\eta_0^2 \frac{\partial^2 w_0}{\partial t^2} = \frac{\partial^2 w_0}{\partial \theta^2} + \frac{\partial \Phi}{\partial \theta} - (1 + a_3)\frac{\partial u_0}{\partial \theta} - \qquad (2)$$
$$-a_3 w_0 + \beta_4 p,$$
$$\gamma_0^2 \frac{\partial^2 \Phi}{\partial t^2} = \frac{\partial^2 \Phi}{\partial \theta^2} - a_2\frac{\partial w_0}{\partial \theta} - a_2\Phi + a_2 u_0,$$

$$\frac{1}{1-\nu_0^2}\frac{\partial^2 u_0}{\partial \theta^2} + \frac{\operatorname{ctg}\theta}{1-\nu_0^2}\frac{\partial u_0}{\partial \theta} + \frac{2(1+\nu_0)k_s + 1 - \nu_0}{2(1-\nu_0^2)k_s}\frac{\partial w_0}{\partial \theta} -$$
$$-\frac{\nu_0 + (1-\nu_0)\cos^2\theta}{(1-\nu_0^2)\sin^2\theta}u_0 + \frac{\Phi}{2(1+\nu_0)k_s} = \gamma_0^2\frac{\partial^2 u_0}{\partial t^2} - q,$$
$$\frac{1}{2(1+\nu_0)k_s}\frac{\partial^2 w_0}{\partial \theta^2} - \frac{1}{1-\nu_0}\frac{\partial u_0}{\partial \theta} + \frac{\operatorname{ctg}\theta}{2(1+\nu_0)k_s}\frac{\partial w_0}{\partial \theta} +$$
$$+\frac{1}{2(1+\nu_0)k_s}\frac{\partial \Phi}{\partial \theta} - \frac{\operatorname{ctg}\theta}{1-\nu_0}u_0 - \frac{2}{1-\nu_0}w_0 +$$
$$+\frac{\operatorname{ctg}\theta}{2(1+\nu_0)k_s}\Phi = \gamma_0^2\frac{\partial^2 w_0}{\partial t^2} - p, \qquad (3)$$
$$\frac{\partial^2 \Phi}{\partial \theta^2} + \operatorname{ctg}\theta\frac{\partial \Phi}{\partial \theta} - \frac{E_0 h R^2}{2(1+\nu_0)k_s D}\frac{\partial w_0}{\partial \theta} -$$
$$-\frac{(1-\nu_0)k_s D(2\nu_0 + (1-\nu_0)\sin 2\theta) + E_0 h R^2 \sin^2\theta}{2(1+\nu_0)k_s D\sin^2\theta}\Phi =$$
$$= \eta_0^2\frac{\partial^2 \Phi}{\partial t^2},$$

where

$$\gamma_0^2 = \left\| \begin{matrix} C_0^2 / C_{02}^2 \\ \rho_0 k_1 C_0^2 / E_0 \end{matrix} \right\|, \quad \eta_0^2 = \left\| \begin{matrix} C_0^2 / C_{01}^2 \\ \rho_0 h^3 C_0^2 k_r / 12D \end{matrix} \right\|,$$

$$C_{01}^2 = \frac{E_0}{(1-\nu_0^2)\rho_0}, \quad C_{02}^2 = \frac{b_1^2 E_0}{2(1+\nu_0)},$$

$$a_2 = \frac{6(1-\nu_0) b_1^2 R^2}{h^2}, \quad a_3 = \frac{2}{(1-\nu_0) b_1^2},$$

$$\beta_3 = \frac{(1-\nu_0^2) K^2 R}{E_0^2 h}, \quad \beta_4 = \frac{2(1+\nu_0) K^2 R}{b_1^2 E_0^2 h},$$

$$b_1^2 = \frac{5}{6}, \quad a_4 = \frac{1}{a_3}, \quad k_1 = 1 + \frac{h^2}{12R^2},$$

$$k_r = 1 + \frac{3h^2}{20R^2}, \quad D = \frac{E_0 h^3}{12(1-\nu_0^2)}, \quad k_s = \frac{5}{6},$$

here Φ – angle of rotation of the normal section to the middle surface, b_1^2 – coefficient that considers the distribution of tangential forces in the transverse section of the cylindrical shell, k_S – shear ratio of the spherical shell, D – cylindrical stiffness, ν_0, E_0, ρ_0 – Poisson's ratio, Young's modulus and density of the shell material, p и q – respectively, the radial and tangential components of the distributed load acting on the shell, R − is the shell radius.

The motion of an elastic medium is described by scalar potential φ and non-zero component of vector potential ψ, which satisfy the wave equations [2 − 5]:

$$\Delta\varphi = \frac{\partial^2 \varphi}{\alpha^2 \partial t^2}, \quad \Delta\psi = \frac{\partial^2 \psi}{\beta^2 \partial t^2},$$
$$\Delta \equiv \left\| \begin{matrix} \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} \\ \dfrac{\partial^2}{\partial r^2} + \dfrac{\partial}{r \partial r} + \dfrac{\partial^2}{\partial z^2} \end{matrix} \right\|. \quad (4)$$

The integrals were calculated using the method of mechanical quadratures, in particular, the symmetric Gregory quadrature formula for equidistant nodes. The Cauchy problem for the differential equation (52) was solved by the Adams method (closed-type formulas) [2 – 6] of order m_1 with a local truncation error $O(\Delta t^{m_1+1})$ [7 – 9, 11]. As a result of discretization, we obtain a system of linear algebraic equations (SLAE). Calculations have shown that with an increase in the reduction size N, the determinant of the SLAE matrix increases indefinitely. The SLAE is poorly defined: as the reduction size N tends to infinity, the value of the determinant of the SLAE matrix also tends to infinity. This is due to the fact that the kernels $Q_{11}(n,t)$, $Q_{22}(n,t)$ in (43), (44) have asymptotic $\exp(O(n))$ in the parameter n, $\tilde{Q}_{11}(n,t)$ and $\tilde{Q}_{22}(n,t)$ in (46) and (47) have asymptotic $O\left(1/n\right)\exp(O(n))$ in the parameter n. Methods of Tikhonov regularization and orthogonal polynomials do not work to neutralize such an exponential singularity. The approach [1 – 5] for solving problems of dynamics, developed by V.D. Kubenko, makes it impossible to study the impact of elastic cylindric and spheric shells of the S.P. Timoshenko type and elastic bodies on an elastic foundation [7 – 9, 11]. In addition, this approach makes it possible to determine the stress-strain state only on the surface of the medium into which the striker penetrates.

CONCLUSIONS

As a result of an attempt to solve the plane and the axisymmetric problems of the impact of a cylindric and a spheric fine shells of the S.P. Timoshenko type on the surface of an elastic half-space, applying the method of V.D. Kubenko, the limitations of this technique were revealed. This technique does not allow solving plane and axisymmetric [1] problems of dynamics for refined shells of the S.P. Timoshenko type and elastic bodies.

To solve [10, 12–15] the problems of impact and nonstationary interaction [16–20], the elastoplastic formulation [21–23] can be used. It should be noted that to calibrate the computational [2] process in the elastoplastic formulation at the elastic stage, it is convenient and expedient to use the technique [2–6] for solving the problems of dynamics, developed by V.D. Kubenko [7–9, 11].

Keywords: impact, elastic, elastic-plastic, half-space, axisymmetric problem, fine, spherical shell, S.P. Timoshenko.

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