One approach to the axisymmetric problem of impact of fine shells of the S.P. Timoshenko type on elastic half-space

Vladislav Bogdanov

Progressive Research Solutions Pty. Ltd.
28/2 Buller Rd, Artarmon, Sydney, Australia 2064
vladislav_bogdanov@hotmail.com, orcid.org/0000-0002-3424-1801

Received 06.02.2021, accepted after revision 06.09.2021
https://doi.org/10.32347/tit2021.42.0302

Abstract. Refined model of S.P. Timoshenko makes it possible to consider the shear and the inertia rotation of the transverse section of the shell. Disturbances spread in the shells of S.P. Timoshenko type with finite speed. Therefore, to study the dynamics of propagation of wave processes in the fine shells of S.P. Timoshenko type is an important aspect as well as it is important to investigate a wave processes of the impact, shock in elastic foundation in which a striker is penetrating. The method of the outcoming dynamics problems to solve an infinite system of integral equations Volterra of the second kind and the convergence of this solution are well studied. Such approach has been successfully used for cases of the investigation of problems of the impact a hard bodies and an elastic fine shells of the Kirchhoff-Love type on elastic a half-space and a layer. In this paper an attempt is made to solve the axisymmetric problem of the impact of an elastic fine spheric shell of the S.P. Timoshenko type on an elastic half-space using the method of the outcoming dynamics problems to solve an infinite system of integral equations Volterra of the second kind. It is shown that this approach is not acceptable for investigated in this paper axisymmetric problem. The discretization using the Gregory methods for numerical integration and Adams for solving the Cauchy problem of the reduced infinite system of Volterra equations of the second kind results in a poorly defined system of linear algebraic equations: as the size of reduction increases the determinant of such a system to aim at infinity. This technique does not allow to solve plane and axisymmetric problems of dynamics for fine shells of the S.P. Timoshenko type and elastic bodies. This shows the limitations of this approach and leads to the feasibility of developing other mathematical approaches and models. It should be noted that to calibrate the computational process in the elastoplastic formulation at the elastic stage, it is convenient and expedient to use the technique of the outcoming dynamics problems to solve an infinite system of integral equations Volterra of the second kind.

Keywords: impact, elastic, elastic-plastic, half-space, axisymmetric problem, fine, spherical shell, S.P. Timoshenko.

Vladislav Bogdanov
Director
PhD phys. and math.
Snr. Res. Ass.

INTRODUCTION

The approach [1 – 5] for solving problems of dynamics, developed in [6 – 8, 10], makes it possible to determine the stress-strain state of elastic half-space and a layer during penetration of absolutely rigid bodies [1, 2, 7, 8, 10] and the stress-strain state of elastic Kirchhoff–Love type fine shells and elastic half-spaces and layers at their collision [3 − 6]. This led to the feasibility of developing other mathematical approaches and models. In [9, 11 – 14], a new approach to solving the problems of impact and nonstationary interaction in the elas-

toplastic mathematical formulation [15 – 19] was developed. In non-stationary problems, the action of the striker is replaced by a distributed load in the contact area, which changes according to a linear law [20 – 22]. The contact area remains constant. The developed elastoplastic formulation makes it possible to solve impact problems when the dynamic change in the boundary of the contact area is considered and based on this the movement of the striker as a solid body with a change in the penetration speed is taken into account. Also, such an elastoplastic formulation makes it possible to consider the hardening of the material in the process of nonstationary and impact interaction.

The solution of problems for elastic shells [23 – 26], elastic half-space [27 – 29], elastic layer [30], elastic rod [31, 32] were developed using method of the influence functions [33]. In [23] the process of non-stationary interaction of an elastic cylindrical shell with an elastic half-space at the so-called "supersonic" stage of interaction is studied. It is characterized by an excess of the expansion rate areas of contact interaction speed of propagation tension-compression waves in elastic half-space. The solution was developed using influence functions corresponding concentrated force or kinematic actions for an elastic isotropic half-space which were found and investigated in [33].

In this paper, we investigate the approach [3 – 6] for solving the axisymmetric problem of the impact of a spherical fine shell of the S.P. Timoshenko type on an elastic half-space.

It is shown that the approach [1 – 4], after the reduction of the infinite system of Volterra integral equations of the second kind [5 – 7, 10] and discretization using the Gregory methods for numerical integration and Adams for solving the Cauchy problem, a poorly defined system of linear algebraic equations is obtained for which the determinant of the matrix of coefficients increases indefinitely with increasing size of reduction.

PROBLEM FORMULATION

A thin elastic spherical shell, moving perpendicular to the surface of the elastic half-space $z \geq 0$, reaches this surface at time t=0. We associate with the shell, as shown in Fig. 1, a movable spherical coordinate system $r'\varphi'\theta$, where φ' – is the longitude of the radius vector r, θ – is the polar angle. The shell penetrates into the elastic medium at a speed $v_T(t)$, $(0 \leq t \leq T)$, the initial penetration rate is $V_0 = v_T(0)$, T – the time during which the shell interacts with the half-space. The shell thickness h is much less than the radius R of the middle surface of the shell ($h/R \leq 0{,}05$).

Let us denote by $u_0(t,\theta)$, $w_0(t,\theta)$, $p(t,\theta)$, $q(t,\theta)$ the tangential and normal displacements of the points of the middle surface of the shell and the radial and tangential components of the distributed external load, which acts on the shell. With the half-space we associate a fixed cylindrical coordinate system $r\varphi z$, the Oz axis is directed deep into the medium, φ – is the polar angle. Angle θ is plotted from the positive direction of the Oz axis. The physical properties of the half-space material are characterized by elastic constants: volumetric expansion module K, shear modulus μ and density ρ. An elastic medium with constants K, μ, ρ will be associated with a hypothetical acoustic medium with the same constants K, ρ, wherein $\mu = 0$. Under C_p, C_S, C_0 we mean the speed of longitudinal and transverse waves in an elastic half-space and the speed of sound

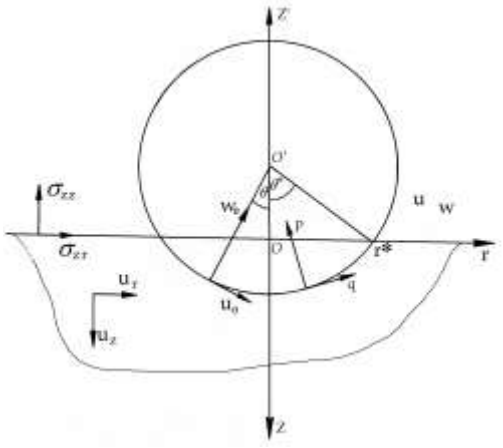

Fig. 1. Scheme of the system spherical shell – half space

in the considered hypothetical acoustic medium.

Let's introduce dimensionless variables:

$$t' = \frac{C_0 t}{R},\ r' = \frac{r}{R},\ z' = \frac{z}{R},\ u_i' = \frac{u_i}{R},$$
$$u_0' = \frac{u_0}{R},\ w_0' = \frac{w_0}{R},\ \sigma_{ij}' = \frac{\sigma_{ij}}{K}, \quad (1)$$
$$v_T' = \frac{v_T}{C_0},\ w_T' = \frac{w_T}{R},\ p' = \frac{p}{KR^2},$$
$$q' = \frac{q}{KR^2},\ M' = \frac{M}{\rho R^3}.\ (i, j = r, \varphi, z)$$
$$\beta^2 = \frac{C_S^2}{C_0^2} = \frac{\mu}{K},\ \alpha^2 = \frac{C_p^2}{C_0^2} = \left(1 + \frac{4\mu}{3K}\right),$$
$$C_0^2 = \frac{K}{\rho},\ b^2 = \frac{\beta^2}{\alpha^2} = \frac{3\mu}{3K + 4\mu}.$$

where $\mathbf{u} = (u_r, u_\varphi, u_z)$ – is the vector of movement of points of the environment; σ_{zz}, σ_{rz} – nonzero components of the stress tensor of the medium; M – is the shell running mass; $v_T(t)$, $w_T(t)$ – speed and movement of the shell as a solid. In what follows, we will use only dimensionless quantities, so we omit the dash. The elastic half-space and the spheric shell are in a state of axisymmetric deformation.

Differential equations (of the S.P. Timoshenko type) describing the dynamics of spherical shells and considering the shear and inertia of rotation of the transverse section, due to (1), take the following form [34, pp. 297, 307]:

$$\frac{1}{1-\nu_0^2}\frac{\partial^2 u_0}{\partial\theta^2} + \frac{\operatorname{ctg}\theta}{1-\nu_0^2}\frac{\partial u_0}{\partial\theta} + \frac{2(1+\nu_0)k_s + 1 - \nu_0}{2(1-\nu_0^2)k_s}\frac{\partial w_0}{\partial\theta} -$$
$$- \frac{\nu_0 + (1-\nu_0)\cos^2\theta}{(1-\nu_0^2)\sin^2\theta}u_0 + \frac{\Phi}{2(1+\nu_0)k_s} = \gamma_0^2\frac{\partial^2 u_0}{\partial t^2} - q,$$
$$\frac{1}{2(1+\nu_0)k_s}\frac{\partial^2 w_0}{\partial\theta^2} - \frac{1}{1-\nu_0}\frac{\partial u_0}{\partial\theta} + \frac{\operatorname{ctg}\theta}{2(1+\nu_0)k_s}\frac{\partial w_0}{\partial\theta} +$$
$$+ \frac{1}{2(1+\nu_0)k_s}\frac{\partial\,\Phi}{\partial\theta} - \frac{\operatorname{ctg}\theta}{1-\nu_0}u_0 - \frac{2}{1-\nu_0}w_0 +$$
$$+ \frac{\operatorname{ctg}\theta}{2(1+\nu_0)k_s}\Phi = \gamma_0^2\frac{\partial^2 w_0}{\partial t^2} - p, \quad (2)$$
$$\frac{\partial^2\Phi}{\partial\theta^2} + \operatorname{ctg}\theta\frac{\partial\Phi}{\partial\theta} - \frac{E_0 hR^2}{2(1+\nu_0)k_s D}\frac{\partial w_0}{\partial\theta} -$$
$$- \frac{(1-\nu_0)k_s D(2\nu_0 + (1-\nu_0)\sin 2\theta) + E_0 hR^2\sin^2\theta}{2(1+\nu_0)k_s D\sin^2\theta}\Phi =$$
$$= \eta_0^2\frac{\partial^2\Phi}{\partial t^2},$$

where

$$\gamma_0^2 = \frac{\rho_0 k_1 C_0^2}{E_0},\ \eta_0^2 = \frac{\rho_0 h^3 C_0^2 k_r}{12D},\ k_1 = 1 + \frac{h^2}{12R^2},$$
$$k_r = 1 + \frac{3h^2}{20R^2},\ D = \frac{E_0 h^3}{12(1-\nu_0^2)},\ k_s = \frac{5}{6},$$

where Φ – angle of rotation of the normal section to the middle surface, k_S – shear ratio, D – cylindrical stiffness, ν_0, E_0, ρ_0 – Poisson's ratio, Young's modulus and density of the shell material, p и q – respectively, the radial and tangential components of the distributed load acting on the shell, R − is the shell radius.

The motion of an elastic medium is described by scalar potential φ and non-zero component of vector potential ψ, which satisfy the wave equations [1 − 4]:

$$\Delta\varphi = \frac{\partial^2\varphi}{\alpha^2\partial t^2},\ \Delta\psi = \frac{\partial^2\psi}{\beta^2\partial t^2},$$
$$\Delta \equiv \frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial r} + \frac{\partial^2}{\partial z^2}. \quad (3)$$

Physical quantities are expressed in terms of wave potentials as follows [5 − 8]:

$$u_r = \frac{\partial\varphi}{\partial r} - \frac{\partial\psi}{\partial z},\ u_z = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial r} + \frac{\psi}{r},$$
$$u_\varphi = 0,\ \sigma_{zz} = (1-2b^2)\frac{\partial^2\varphi}{\partial t^2} +$$
$$+2\beta^2\left(\frac{\partial^2\varphi}{\partial z^2} + \frac{\partial^2\psi}{\partial r\partial z} + \frac{\partial\psi}{r\partial z}\right), \quad (4)$$
$$\sigma_{r\varphi} = \sigma_{\varphi z} = 0,$$
$$\sigma_{rz} = 2\beta^2\frac{\partial^2\varphi}{\partial r\partial z} + \frac{\partial^2\psi}{\partial t^2} - 2\beta^2\frac{\partial^2\psi}{\partial z^2},$$
$$\sigma_{rr} = (1-2b^2)\frac{\partial^2\varphi}{\partial t^2} + 2\beta^2\left(\frac{\partial^2\varphi}{\partial r^2} - \frac{\partial^2\psi}{\partial r\partial z}\right),$$

If the shear modulus μ is set equal to zero $\mu = 0$, then the equations of motion of the elastic medium will be the equations of acoustics.

Let us consider the initial stage of the process of impact of elastic shells on the surface of an elastic half-space [3 − 6], when no plastic deformations occur and the depth of the shell penetration into the medium is small.

The problem of interaction of elastic shells with an elastic half-space is solved in a linear formulation, therefore, we linearize the boundary conditions [1, 2, 7, 8, 10]: we transfer the boundary conditions from the perturbed surface to the undisturbed surface of the bodies that are deformed. We assume that there is no friction between the elastic half-space and the penetrating body, or the slippage condition is valid.

As can be seen from Fig. 1, the projections of the functions u_0, w_0, p and q on the *or* and *oz* axes will be equal:

$$\begin{aligned}
&\mathrm{pr}_{Z'} w_0(t,\theta) = w_0(t,\theta)\cos\theta,\\
&\mathrm{pr}_{Z'} u_0(t,\theta) = u_0(t,\theta)\sin\theta,\\
&\mathrm{pr}_{Z'} p(t,\theta) = p(t,\theta)\cos\theta,\\
&\mathrm{pr}_{Z'} q(t,\theta) = q(t,\theta)\sin\theta,\\
&\mathrm{pr}_{r} w_0(t,\theta) = -w_0(t,\theta)\sin\theta,\\
&\mathrm{pr}_{r} u_0(t,\theta) = u_0(t,\theta)\cos\theta,\\
&\mathrm{pr}_{r} p(t,\theta) = -p(t,\theta)\sin\theta,\\
&\mathrm{pr}_{r} q(t,\theta) = q(t,\theta)\cos\theta.
\end{aligned}$$

Then, in the *zor* coordinate system, the displacements u_z, u_r and stresses σ_{zz} and σ_{rz} at the surface points of the contact area will be written as:

$$\begin{aligned}
&u_z(t,r,0) = w_T(t) - f(r) - w_0(t,\theta)\cos\theta - \\
&-u_0(t,\theta)\sin\theta,\\
&u_r(t,r,0) = -w_0(t,\theta)\sin\theta + \\
&+u_0(t,\theta)\cos\theta,\\
&\sigma_{zz}(t,r,0) = -p(t,\theta)\cos\theta - \\
&-q(t,\theta)\sin\theta,\\
&\sigma_{rz}(t,r,0) = -p(t,\theta)\sin\theta + \\
&+q(t,\theta)\cos\theta,\\
&p(t,\theta) = -\sigma_{zz}(t,r,0)\cos\theta - \sigma_{rz}(t,r,0)\sin\theta, \quad |\theta| < \theta^*,
\end{aligned} \tag{5}$$

$$q(t,\theta) = -\sigma_{zz}(t,r,0)\sin\theta + \sigma_{rz}(t,r,0)\cos\theta, \quad |\theta| < \theta^*,$$

where $w_T(t)$ – displacement of the shell as a rigid body, the function $f(x)$ describes the shell profile, $2\theta^*$ as can be seen from Figure 1, the size of the shell sector in contact with the half-space. In the case of a spherical shell:

$$f(r) = 1 - \sqrt{1 - r^2}.$$

The kinematic condition that determines the half-size of the contact area $x^*(t)$ is written as follows:

$$\begin{aligned}
&w_T(t) - f(r) - u_z(t,r,0) - w_0(t,\theta)\cos\theta - \\
&-u_0(t,\theta)\sin\theta = \begin{cases} 0, & \text{if} \quad r \le r^*(t), \\ \varepsilon < 0, & \text{if} \quad r > r^*(t), \end{cases}
\end{aligned}$$

We assume that the contact area is simply connected region, and this statement is equivalent to the fact that the stresses normal to the contact area are compressive:

$$\sigma_{zz}\big|_{z=0} < 0, \quad r < r^*(t).$$

Based on (4), the boundary conditions in the absence of friction in the contact zone can be formulated as follows:

$$\begin{aligned}
&\left.\frac{\partial u_z}{\partial t}\right|_{z=0} \equiv V(t,r) = v_T(t) - \frac{\partial w_0(t,\theta)}{\partial t}\cos\theta - \\
&-\frac{\partial u_0(t,\theta)}{\partial t}\sin\theta, \quad r < r^*(t),\\
&\sigma_{zz}\big|_{z=0} = 0; \quad r > r^*(t), \quad \sigma_{zr}\big|_{z=0} = 0, \quad r > 0.
\end{aligned} \tag{8}$$

The initial conditions for potentials and – are zero:

$$\varphi\big|_{t=0} = \left.\frac{\partial\varphi}{\partial t}\right|_{t=0} = 0, \quad \psi\big|_{t=0} = \left.\frac{\partial\psi}{\partial t}\right|_{t=0} = 0. \tag{9}$$

For the problem of impact of an elastic shell on an elastic half-space, the velocity and displacement of the impacting body are found from the equation of motion by integrating it.

The equation of motion of a shell of mass M for the problem of impact with an initial velocity V_0 has the form:

$$M\frac{d^2w_T(t)}{dt^2} = -P(t), \quad (10)$$

$$v_T(t)\big|_{t=0} = V_0, \quad w_T(t)\big|_{t=0} = 0, \quad (11)$$

$$P(t) = -2\pi\int_0^{r^*(t)} r\sigma_{zz}(t,r,0)dr. \quad (12)$$

The condition for the absence of disturbances ahead of the front of longitudinal waves and the condition for damping of disturbances at infinity are valid.

$$\varphi\big|_{\rho_1>\alpha t+C_\alpha} = 0, \quad \psi\big|_{\rho_1>\alpha t+C_\alpha} = 0, \quad (13)$$

$$\varphi\big|_{\rho_1\to\infty} \to 0, \quad \psi\big|_{\rho_1\to\infty} \to 0, \quad (14)$$

where $\rho_1 = r^2 + z^2$, $C_\alpha = \text{const}$.

SOLUTION ALGORITHM

Since the impact process is short-term, the perturbation region at each moment of time t is finite. Restricting ourselves to a finite interval of interaction time $(0 \le t \le T)$, it is possible to select a region of a half-space, which by the time moment T covers the entire zone of disturbances. From this point of view, for times $(0 \le t \le T)$, the elastic half-space can be replaced by an elastic half-cylinder $(r \le l;\ z \ge 0)$, the boundaries of which do not reach the perturbations by the time T.

$$l = \alpha T + r^*(T).$$

Thus, for times $(0 \le t \le T)$, the considered problem is reduced to a nonstationary problem for a half-cylinder with mixed boundary conditions at its end. To represent the displacement vector as:

$$u = \text{grad}\varphi + \text{rot}\psi, \quad \text{div}\psi = 0,$$

on the lateral surface of the half-cylinder, we select, for example, the conditions for sliding termination:

$$u_r\big|_{r=l} = 0, \quad \sigma_{zr}\big|_{r=l} = 0, \quad (15)$$

or

$$u_z\big|_{r=l} = 0, \quad \sigma_{rr}\big|_{r=l} = 0. \quad (16)$$

Consider the initial - boundary value problem (2), (3), (8) – (11). Let us represent the normal $w_0(t,\theta)$ and tangential $u_0(t,\theta)$ displacements of the points of the middle surface of the shell and the radial $p(t,\theta)$ and tangential $q(t,\theta)$ components of the distributed external load acting on the shell in the form of series in Legendre polynomials and their derivatives.

$$w_0(t,\theta) = \sum_{n=0}^{\infty} w_{0n}(t)P_n(\cos\theta), \quad (17)$$

$$u_0(t,\theta) = \sum_{n=1}^{\infty} u_{0n}(t)P_n^1(\cos\theta), \quad (18)$$

$$p(t,\theta) = \sum_{n=0}^{\infty} p_n(t)P_n(\cos\theta), \quad (19)$$

$$q(t,\theta) = \sum_{n=1}^{\infty} q_n(t)P_n^1(\cos\theta), \quad (20$$

$$\Phi(t,\theta) = \sum_{n=1}^{\infty} \Phi_n(t)P_n^1(\cos\theta). \quad (21)$$

In the space of Laplace transformants with parameter s, the transformants of functions Φ, w_0 , u_0 , p , q will, due to (17) – (21), have the form:

$$w_0^L(s,\theta) = \sum_{n=0}^{\infty} w_{0n}^L(s)P_n(\cos\theta), \quad (22)$$

$$u_0^L(s,\theta) = \sum_{n=1}^{\infty} u_{0n}^L(s)P_n^1(\cos\theta), \quad (23)$$

$$p^L(s,\theta) = \sum_{n=0}^{\infty} p_n^L(s)P_n(\cos\theta), \quad (24)$$

$$q^L(s,\theta) = \sum_{n=1}^{\infty} q_n^L(s)P_n^1(\cos\theta), \quad (25)$$

$$\Phi^L(s,\theta)=\sum_{n=1}^{\infty}\Phi_n^L(s)P_n^1(\cos\theta), \qquad (26)$$

We apply to the system of equations (2) the Laplace transform in the variable t with the parameter s and substitute their equalities (22) – (26). Equating the coefficients at the same $P_n(\cos\theta)$ and $P_n^1(\cos\theta)$ we obtain the relations connecting the components of the expansion into series of functions Φ^L, w_0^L, u_0^L, p^L and q^L.

$$w_{0,0}^L(s)=\frac{p_0^L(s)}{\gamma_0^2 s^2+2/(1-\nu_0)}, \qquad (27)$$

$$w_{0,n}^L(s)=Q_{11}^L(n,s)p_n^L(s)+Q_{12}^L(n,s)q_n^L(s), \qquad (28)$$

$$u_{0,n}^L(s)=Q_{21}^L(n,s)p_n^L(s)+Q_{22}^L(n,s)q_n^L(s), \qquad (29)$$

$$\Phi_n^L(s)=Q_{31}^L(n,s)p_n^L(s)+Q_{32}^L(n,s)q_n^L(s), \qquad (30)$$

where

$$Q_{ij}^L(n,s)=\frac{\Delta_{ij}(s)}{\Delta(s)},\ (i=1,2,3;\ j=1,2;\ n=\overline{1,\infty}),$$

$$\Delta_{11}(n,s)=\left(\frac{n(n+1)}{1-\nu_0^2}-\frac{1}{1-\nu_0}+\gamma_0^2 s^2\right)\times$$
$$\times\left(n(n+1)-1+\nu_0+R_R+\eta_0^2 s^2\right),$$

$$\Delta_{21}(n,s)=\left(R_R+\left(\frac{2(1+\nu_0)k_s}{1-\nu_0}+1\right)(n(n+1)-\right.$$
$$\left.-1+\nu_0+R_R)+\eta_0^2 s^2\right)\Big/\left(2(1+\nu_0)Dk_s\right),$$

$$\Delta_{12}(n,s)=\frac{n(n+1)}{1-\nu_0}\left(n(n+1)-1+\nu_0+R_R+\eta_0^2 s^2\right),$$

$$\Delta_{22}(n,s)=-\frac{n(n+1)R_R}{2(1+\nu_0)k_s}+\left(\frac{n(n+1)}{2(1+\nu_0)k_s}+\right.$$
$$\left.+\frac{2}{1-\nu_0}+\gamma_0^2 s^2\right)\left(n(n+1)-1+\nu_0+R_R+\eta_0^2 s^2\right),$$

$$\Delta_{31}(n,s)=\frac{R_R}{h}\left(-n(n+1)+\frac{1}{1+\nu_0}+\gamma_0^2 s^2\right),$$

$$\Delta_{32}(n,s)=-\frac{n(n+1)R_R}{(1-\nu_0)h},\quad R_R=\frac{R^2E_0h}{2(1+\nu_0)Dk_s},$$

$$\Delta(s)=-\eta_0^2\gamma_0^4\left(s^6+\bar{A}_a s^4+\bar{B}_b s^2+\bar{C}_c\right),$$

$$\bar{A}_a=\frac{1}{\gamma_0^2}\left(\frac{n(n+1)}{1-\nu_0^2}+\frac{n(n+1)}{2(1+\nu_0)k_s}+\frac{1}{1+\nu_0}+\right.$$
$$\left.+\frac{2}{1-\nu_0}\right)+\frac{1}{\eta_0^2}\left(n(n+1)-1+\nu_0+\frac{R^2E_0h}{2(1+\nu_0)Dk_s}\right),$$

$$\bar{B}_b=\frac{1}{\eta_0^2\gamma_0^4}\left(\eta_0^2\left(\frac{n(n+1)}{1-\nu_0^2}-\frac{1}{1+\nu_0}\right)\left(\frac{n(n+1)}{2(1+\nu_0)k_s}+\right.\right.$$
$$\left.+\frac{2}{1-\nu_0}\right)+\gamma_0^2\left(\frac{n(n+1)}{1-\nu_0^2}+\frac{n(n+1)}{2(1+\nu_0)k_s}+\frac{1}{1+\nu_0}+\right.$$
$$\left.\left.+\frac{2}{1-\nu_0}\right)\left(n(n+1)-1+\nu_0+\frac{R^2E_0h}{2(1+\nu_0)k_s}+\frac{2}{1-\nu_0}\right)-\right.$$
$$-\frac{n(n+1)}{2(1+\nu_0)Dk_s}\left(\frac{\eta_0^2}{(1-\nu_0)^2}\left(2(1+\nu_0)k_s+1-\nu_0\right)+\right.$$
$$\left.\left.+\frac{\gamma_0^2R^2E_0h}{2(1+\nu_0)Dk_s}\right)\right),$$

$$\bar{C}_c=\frac{1}{\eta_0^2\gamma_0^4}\left(\left(\frac{n(n+1)}{1-\nu_0^2}-\frac{1}{1+\nu_0}\right)\left(\left(\frac{n(n+1)}{2(1+\nu_0)k_s}+\right.\right.\right.$$
$$\left.+\frac{2}{1-\nu_0}\right)\left(n(n+1)-1+\nu_0+\frac{R^2E_0h}{2(1+\nu_0)Dk_s}\right)-$$
$$\left.-\frac{n(n+1)R^2E_0h}{4(1+\nu_0)^2Dk_s^2}\right)+\frac{n(n+1)}{2(1-\nu_0^2)k_s}\times$$
$$\times\left(\frac{R^2E_0h}{2(1+\nu_0)Dk_s}-\left(\frac{2(1+\nu_0)k_s}{1-\nu_0}+1\right)\times\right.$$
$$\left.\left.\times\left(n(n+1)-1+\nu_0+\frac{R^2E_0h}{2(1+\nu_0)Dk_s}\right)\right)\right).$$

Then applying the inverse Laplace transform to (27) – (30), by the theorem on the

convolution of the originals of two functions, we have:

$$\dot{w}_{0,0}(t)=\frac{1}{\gamma_0^2}\int_0^t p_0(\tau)\cos\left(\frac{t-\tau}{\gamma_0\sqrt{(1-\nu_0)/2}}\right)d\tau, \quad (31)$$

$$\dot{w}_{0,n}(t)=\int_0^t p_n(\tau)Q_{11}(n,t-\tau)d\tau+ \\ +\int_0^t q_n(\tau)Q_{12}(n,t-\tau)d\tau, \quad (32)$$

$$\dot{u}_{0,n}(t)=\int_0^t p_n(\tau)Q_{21}(n,t-\tau)d\tau+ \\ +\int_0^t q_n(\tau)Q_{22}(n,t-\tau)d\tau, \quad (33)$$

$$\dot{\Phi}_n(t)=\int_0^t p_n(\tau)Q_{31}(n,t-\tau)d\tau+ \\ +\int_0^t q_n(\tau)Q_{32}(n,t-\tau)d\tau,\ \ (n=\overline{1,\infty}), \quad (34)$$

where

$$Q_{ij}(n,t)=4\Big[(\Delta_r R_{ij}+\Delta_i I_{ij})\mathrm{ch}(r_0 t)\cos(\sigma_0 t)+ \\ +(\Delta_i R_{ij}-\Delta_r I_{ij})\mathrm{sh}(r_0 t)\sin(\sigma_0 t)\Big]\Big/\left(\Delta_r^2+\Delta_i^2\right)+ \\ +\frac{2\Delta_{ij}(n,s_1^2)\left(\mathrm{H}(s_1^2)\mathrm{ch}(s_1 t)+\mathrm{H}(-s_1^2)\cos(s_1 t)\right)}{\Delta'(s_1^2)},$$

where H(x) – Heaviside function,

$$r_0=(r^2+\sigma^2)^{1/4}\cos(\varphi/2),$$

$$\sigma_0=(r^2+\sigma^2)^{1/4}\sin(\varphi/2),\ \ \varphi=\mathrm{arctg}(\sigma/r),$$

$$r=-((A+B)/2+\bar{A}_a/3),\ \ \sigma=\sqrt{3}(A-B)/2,$$

$$s_1^2=A+B-\bar{A}_a/3,\ \ A=(-q'/2+Q^{1/2})^{1/3},$$

$$B=(-q'/2-Q^{1/2})^{1/3},\ \ Q=(p'/3)^3+(q'/2)^2,$$

$$q'=2(\bar{A}_a/3)^3-\bar{A}_a\bar{B}_b/3+\bar{C}_c,$$

$$p'=-\bar{A}_a^2/3+\bar{B}_b,\ \ r_1=r^2-\sigma^2,\ \ \sigma_1=2r\sigma,$$

$$R_{11}=\eta_0^2\gamma_0^2 r_1+\left(\eta_0^2\left(\frac{n(n+1)}{1-\nu_0^2}-\frac{1}{1-\nu_0}\right)+\right. \\ \left.+\gamma_0^2(n(n+1)-1+\nu_0+R_R)\right)r+ \\ +\left(\frac{n(n+1)}{1-\nu_0^2}-\frac{1}{1-\nu_0}\right)(n(n+1)-1+\nu_0+R_R),$$

$$I_{11}=\eta_0^2\gamma_0^2\sigma_1+\left(\eta_0^2\left(\frac{n(n+1)}{1-\nu_0^2}-\frac{1}{1-\nu_0}\right)+\right. \\ \left.+\gamma_0^2(n(n+1)-1+\nu_0+R_R)\right)\sigma,$$

$$R_{12}=\frac{n(n+1)}{1-\nu_0}(n(n+1)-1+\nu_0+R_R+\eta_0^2 r),$$

$$I_{12}=\eta_0^2\frac{n(n+1)}{1-\nu_0}\sigma,$$

$$R_{21}=\frac{1}{2(1+\nu_0)Dk_s}\left(\left(\frac{2(1+\nu_0)k_s}{1-\nu_0}+1\right)\times\right. \\ \left.\times(n(n+1)-1+\nu_0+R_R+\eta_0^2 r)-R_R\right),$$

$$I_{21}=\eta_0^2\left(\frac{1}{1-\nu_0}+\frac{1}{2(1+\nu_0)k_s}\right)\sigma,$$

$$R_{22}=\eta_0^2\gamma_0^2 r_1+\left(\eta_0^2\left(\frac{n(n+1)}{2(1+\nu_0)k_s}+\frac{2}{1-\nu_0}\right)+\right. \\ \left.+\gamma_0^2(n(n+1)-1+\nu_0+R_R)\right)r+ \\ +\left(\frac{n(n+1)}{2(1+\nu_0)k_s}+\frac{2}{1-\nu_0}\right)(n(n+1)-1+\nu_0+R_R)- \\ -\frac{n(n+1)R_R}{2(1+\nu_0)k_s},$$

$$I_{22}=\eta_0^2\gamma_0^2\sigma_1+\left(\eta_0^2\left(\frac{n(n+1)}{2(1+\nu_0)k_s}+\frac{2}{1-\nu_0}\right)+\right. \\ \left.+\gamma_0^2(n(n+1)-1+\nu_0+R_R)\right)\sigma,$$

$$R_{31}=\frac{R_R}{h}\left(-n(n+1)+\frac{1}{1-\nu_0}+\gamma_0^2 r\right),$$

$$I_{31}=\gamma_0^2\frac{R_R}{h}\sigma,\ \ R_{32}=-\frac{R_R}{h}n(n+1),\ \ I_{32}=0,$$

$$\Delta'(s)=-\eta_0^2\gamma_0^4\left[6s^4+4\bar{A}_a s^2+2\bar{B}_b\right],$$

$$\Delta_r=-\eta_0^2\gamma_0^4\left[6r_1+4\bar{A}_a r+2\bar{B}_b\right],$$

$$\Delta_i=-\eta_0^2\gamma_0^4\left[6\sigma_1+4\bar{A}_a\sigma\right].$$

$$w_{0,0}(t)=\frac{1}{\gamma_0}\int_0^t p_0(\tau)\sin\left(\frac{t-\tau}{\gamma_0\sqrt{(1-\nu_0)/2}}\right)d\tau,$$

$$w_{0,n}(t)=\int_0^t p_n(\tau)\tilde{Q}_{11}(n,t-\tau)d\tau+ \\ +\int_0^t q_n(\tau)\tilde{Q}_{12}(n,t-\tau)d\tau, \quad (35)$$

$$u_{0,n}(t)=\int_0^t p_n(\tau)\tilde{Q}_{21}(n,t-\tau)d\tau+ \qquad (36)$$

$$+\int_0^t q_n(\tau)\tilde{Q}_{22}(n,t-\tau)d\tau,$$

$$\Phi_n(t)=\int_0^t p_n(\tau)\tilde{Q}_{31}(n,t-\tau)d\tau+$$

$$+\int_0^t q_n(\tau)\tilde{Q}_{32}(n,t-\tau)d\tau,\ \ (n=\overline{1,\infty}),$$

where

$$\tilde{Q}_{ij}(n,t)=4\Big[(\delta_r R_{ij}+\delta_i I_{ij})\mathrm{sh}(r_0 t)\cos(\sigma_0 t)+$$

$$+(\delta_i R_{ij}-\delta_r I_{ij})\mathrm{ch}(r_0 t)\sin(\sigma_0 t)\Big]\Big/\left(\delta_r^2+\delta_i^2\right)+$$

$$+\frac{2\Delta_{ij}(n,s_1^2)\left(\mathrm{H}(s_1^2)\mathrm{sh}(s_1 t)+\mathrm{H}(-s_1^2)\sin(s_1 t)\right)}{(s_1\Delta'(s_1^2))},$$

$$\delta_r=r_0\Delta_r-\sigma_0\Delta_i,\ \ \delta_i=\sigma_0\Delta_r+r_0\Delta_i.$$

We apply to the system of equations (2) the Laplace transform in the variable t (s is the transformation parameter) and the Fourier method of separation of variables, considering the evenness in x of the potential and the oddness of the potential , and require the satisfaction of condition (13) – (14). Then, in the space of Laplace transformants, we obtain the following representations for wave potentials [5]:

$$\varphi^L(s,r,z)=\sum_{n=0}^{\infty}A_n(s)\exp\left(-z\sqrt{\frac{s^2}{\alpha^2}+\lambda_n^2}\right)J_0(\lambda_n r),$$

$$\psi^L(s,r,z)=\sum_{n=0}^{\infty}B_n(s)\exp\left(-z\sqrt{\frac{s^2}{\beta^2}+\lambda_n^2}\right)J_1(\lambda_n r), \qquad (37)$$

where λ_n – the eigenvalues of the problem, which are determined from conditions (15) taking into account (4) and are the roots of the equality

$$J_1(\lambda_n l)=0,\quad (n=\overline{0,\infty}).$$

In (37) $A_n(s)$ and $B_n(s)$ are determined from the boundary conditions. It follows from representations (37) and relations (4) that the sought-for functions on the surface of a half-space are represented as series in the system of eigenfunctions of the problem.

$$u_z(t,r,0)=\sum_{n=0}^{\infty}u_{zn}(t)J_0(\lambda_n r),$$

$$u_r(t,r,0)=\sum_{n=1}^{\infty}u_{rn}(t)J_1(\lambda_n r),$$

$$\sigma_{zz}(t,r,0)=\sum_{n=0}^{\infty}\sigma_{zn}(t)J_0(\lambda_n r),$$

$$\sigma_{zr}(t,x,0)=\sum_{n=1}^{\infty}\sigma_{zrn}(t)J_1(\lambda_n r).$$

Just as in [1 – 5], the dependence between the harmonics of the vertical component of the velocity and normal stresses on the surface of the half-space is determined [6 – 8, 10]:

$$\sigma_{zn}(t)=-\alpha\left(V_n(t)+\int_0^t V_n(\tau)F(t-\tau)d\tau\right), \qquad (38)$$

where

$$F_n(t)=-\alpha\lambda_n J_1(\alpha\lambda_n t)+2b\beta\lambda_n\left\{\beta^2\lambda_n^2 t^2(\bar{J}_0(\alpha\lambda_n t)-\right.$$

$$-\bar{J}_0(\beta\lambda_n t)-J_1(\alpha\lambda_n t)+J_1(\beta\lambda_n t))+\beta\lambda_n\ t\times$$

$$\times(bJ_0(\alpha\lambda_n t)-J_0(\beta\lambda_n t))+(2-b^2)\bar{J}_0(\alpha\lambda_n t)-$$

$$\left.-\bar{J}_0(\beta\lambda_n t)\right\},$$

where $J_0(t)$, $J_1(t)$ – Bessel functions of the first kind of zero and first order, respectively, and the function $\bar{J}_0(t)$ is defined as follows:

$$\bar{J}_0(t)=\int_0^t J_0(\tau)d\tau.$$

Further, we will satisfy the mixed boundary conditions (8). From (8), (38) we obtain the following representation for the vertical component of the velocity on the surface of the half-space:

$$\sum_{n=0}^{\infty}V_n(t)J_0(\lambda_n r)=\mathrm{H}(r^*-r)\times$$

$$\left\{v_T(t)-\dot{w}_0(t,\theta)\cos\theta-\dot{u}_0(t,\theta)\sin\theta\right\}- \qquad (39)$$

$$-\mathrm{H}(r-r^*)\sum_{n=0}^{\infty}J_0(\lambda_n r)\int_0^t V_n(\tau)F_n(t-\tau)d\tau.$$

Substituting (22) and (23) into (39) with allowance for $r=\sin\theta$, arising from geometric considerations in the zone of the contact region, and representing both parts of (39) in the form of series in $J_0(\lambda_n r)$, we obtain an infinite system of Volterra integral equations (ISVIE) of the second kind regarding to unknown harmonics velocity on the surface of the half-space $(n=\overline{0,\infty})$:

$$V_n(t)+\sum_{m=0}^{\infty}\alpha_{mn}^{(4)}(r^*)\int_0^t V_m(\tau)F_m(t-\tau)d\tau+$$
$$+\sum_{m=0}^{\infty}\left[\alpha_{mn}^{(5)}(r^*)\dot{w}_{0m}(t)+\alpha_{mn}^{(6)}(r^*)\dot{u}_{0m}(t)\right]\times$$
$$\times\int_0^t V_m(\tau)F_m(t-\tau)d\tau=C_n(r^*)v_T(t),$$

where

$$\alpha_{mn}^{(4)}(r^*)=\frac{1}{N_n^2}\int_{r^*}^{l} rJ_0(\lambda_m r)J_0(\lambda_n r)dr,$$

$$\alpha_{mn}^{(5)}(r^*)=\frac{1}{N_n^2}\int_0^{r^*} r\sqrt{1-r^2}P_m\left(\sqrt{1-r^2}\right)J_0(\lambda_n r)dr,$$

$$\alpha_{mn}^{(6)}(r^*)=\frac{1}{N_n^2}\int_0^{r^*} r^2\sqrt{1-r^2}\frac{\partial}{\partial r}P_m\left(\sqrt{1-r^2}\right)J_0(\lambda_n r)dr,$$

$$C_n(r^*)=\frac{1}{N_n^2}\int_0^{r^*} rJ_0(\lambda_n r)dr,\quad N_n^2=\int_0^{l} r\left(J_0(\lambda_n r)\right)^2 dr.$$

The functions $\dot{w}_{0m}(t)$, $\dot{u}_{0m}(t)$ and $\dot{\Phi}_n(t)$ are determined from relations (31) – (34), but they involve unknown functions $p_n(t)$ and $q_n(t)$. Let us deal with their exclusion, for this we use conditions (6), (7), which can be rewritten using (38) in the form:

$$\sum_{n=0}^{\infty}p_n(t)P_n(\cos\theta)=\alpha\,\mathrm{H}(\theta^*-|\theta|)\cos\theta\times$$
$$\times\sum_{n=0}^{\infty}J_0(\lambda_n\sin\theta)\left(V_n(t)+\int_0^t V_n(\tau)F_n(t-\tau)d\tau\right),$$
$$\sum_{n=0}^{\infty}q_n(t)P_n^1(\sin\theta)=\alpha\,\mathrm{H}(\theta^*-|\theta|)\sin\theta\times$$
$$\times\sum_{n=0}^{\infty}J_0(\lambda_n\sin\theta)\left(V_n(t)+\int_0^t V_n(\tau)F_n(t-\tau)d\tau\right).$$

Using the orthogonality of the polynomials and the associated Legendre polynomials, we obtain the relations establishing the relationship between the harmonics of the series expansions of the functions *p*, *q* and *V*:

$$p_n(t)=\sum_{m=0}^{\infty}\gamma_{mn}^{(3)}(\theta^*)\left(V_m(t)+\int_0^t V_m(\tau)F_m(t-\tau)d\tau\right),$$
$$q_n(t)=\sum_{m=0}^{\infty}\gamma_{mn}^{(4)}(\theta^*)\left(V_m(t)+\int_0^t V_m(\tau)F_m(t-\tau)d\tau\right),$$

where

$$\gamma_{mn}^{(3)}(\theta^*)=\frac{\alpha}{\bar{N}_n^2}\int_0^{\theta^*}\cos\theta\sin\theta P_n(\cos\theta)J_0(\lambda_m\sin\theta)d\theta,$$

$$\gamma_{mn}^{(4)}(\theta^*)=\frac{\alpha}{\tilde{N}_n^2}\int_0^{\theta^*}\sin^2\theta P_n^1(\cos\theta)J_0\left(\lambda_m\sin\theta\right)d\theta,$$

$$\bar{N}_n^2=\int_0^{\pi}\sin\theta\left(P_n(\cos\theta)\right)^2 d\theta,$$

$$\tilde{N}_n^2=\int_0^{\pi}\sin\theta\left(P_n^1(\cos\theta)\right)^2 d\theta.$$

Thus, the final form of the resolving ISVIE of the second kind will be as follows:

$$V_n(t)+\sum_{m=0}^{\infty}\alpha_{mn}^{(4)}(r^*)\int_0^t V_m(\tau)F_m(t-\tau)d\tau+$$
$$+\sum_{m=0}^{\infty}\alpha_{mn}^{(5)}(r^*)\sum_{k=0}^{\infty}\int_0^t\gamma_{km}^{(3)}(\theta^*(\tau))\Big(V_k(\tau)+$$
$$+\int_0^{\tau}V_k(\xi)F_k(\tau-\xi)d\xi\Big)Q_{11}(m,t-\tau)d\tau+$$
$$+\sum_{m=0}^{\infty}\alpha_{mn}^{(5)}(r^*)\sum_{k=0}^{\infty}\int_0^t\gamma_{km}^{(4)}(\theta^*(\tau))\Big(V_k(\tau)+$$
$$+\int_0^{\tau}V_k(\xi)F_k(\tau-\xi)d\xi\Big)Q_{12}(m,t-\tau)d\tau+$$
$$+\sum_{m=0}^{\infty}\alpha_{mn}^{(6)}(r^*)\sum_{k=0}^{\infty}\int_0^t\gamma_{km}^{(3)}(\theta^*(\tau))\Big(V_k(\tau)+$$

$$+\int_0^{\tau} V_k(\xi)F_k(\tau-\xi)d\xi\Big)Q_{21}(m,t-\tau)d\tau+$$
$$+\sum_{m=0}^{\infty}\alpha_{mn}^{(6)}(r^*)\sum_{k=0}^{\infty}\int_0^t\gamma_{km}^{(4)}(\theta^*(\tau))\Big(V_k(\tau)+$$
$$+\int_0^{\tau} V_k(\xi)F_k(\tau-\xi)d\xi\Big)Q_{22}(m,t-\tau)d\tau= \quad (40)$$
$$=C_n(r^*)v_T(t),\quad (n=\overline{0,\infty}).$$

To solve the problem, when the shell penetration velocity $v_T(t)$ is a predetermined function, it is sufficient to numerically implement equations (40).

The expression for the reaction force of the elastic half-space (12), using (38), can be rewritten as:

$$P(t)=-2\pi\int_0^{r^*(t)} r\sigma_{zz}(t,r,0)dr=\alpha\pi r^*(t)\times$$
$$\times\left\{v_T(t)r^*(t)+2\sum_{n=0}^{\infty}\frac{J_1\left(\lambda_n r^*\right)}{\lambda_n}\int_0^t V_n(\tau)F_n(t-\tau)d\tau\right\}.$$

The equation of motion of the shell (10) with the initial conditions takes the form:

$$M\frac{dv_T(t)}{dt}=-\alpha\pi r^*(t)\Big\{v_T(t)r^*(t)+$$
$$+2\sum_{n=0}^{\infty}\frac{J_n\left(\lambda_n r^*\right)}{\lambda_n}\int_0^t V_n(\tau)F_n(t-\tau)d\tau\Big\}. \quad (41)$$

To solve the problem of impact with an initial velocity V_0, the system of equations (40) must be supplemented with the equation of motion (41).

The contact area is determined considering the rise of the medium from the condition:

$$\delta_{1j}v_T t+\delta_{2j}\int_0^t v_T(\tau)d\tau-f(r^*)-$$
$$-\sum_{n=0}^{\infty}J_0\left(\lambda_n r^*\right)\int_0^t V_n(\tau)d\tau- \quad (42)$$
$$-\sqrt{1-r^{*2}}\sum_{n=0}^{\infty}P_n\left(\sqrt{1-r^{*2}}\right)\times$$
$$\times\sum_{m=0}^{\infty}\int_0^t\Big[\gamma_{mn}^{(3)}(\arcsin r^*(\tau))\tilde{Q}_{11}(n,t-\tau)+$$
$$+\gamma_{mn}^{(4)}(\arcsin r^*(\tau))\tilde{Q}_{12}(n,t-\tau)\Big]\times$$
$$\times\left(V_m(\tau)+\int_0^{\tau}V_m(\xi)F_m(\tau-\xi)d\xi\right)d\tau-$$
$$-r^*\sqrt{1-r^{*2}}\sum_{n=0}^{\infty}P_n^1\left(\sqrt{1-r^{*2}}\right)\times$$
$$\times\sum_{m=0}^{\infty}\int_0^t\Big[\gamma_{mn}^{(3)}(\arcsin r^*(\tau))\tilde{Q}_{21}(n,t-\tau)+$$
$$+\gamma_{mn}^{(4)}(\arcsin r^*(\tau))\tilde{Q}_{22}(n,t-\tau)\Big]\times$$
$$\times\left(V_m(\tau)+\int_0^{\tau}V_m(\xi)F_m(\tau-\xi)d\xi\right)d\tau=$$
$$=\begin{cases}0, & \text{if } r<r^*(t)\\ \varepsilon<0, & \text{if } r>r^*(t)\end{cases}.$$

where $\delta_{ij}=\{0,\ \text{if}\ i\neq j;\ 1,\ \text{if}\ i=j\}$ – Kronecker symbol. Index j=1 corresponds to the case when the body penetrates into the medium at a speed varying according to a predetermined law (setting 1); if the velocity of the penetrating body is known only at the initial moment of time $t=0$, and at subsequent moments is determined from the equation of motion (statement 2), then j=2. If we exclude the fourth term in relation (42), then we obtain a condition from which the boundary of the contact region is determined without considering the rise of the medium.

NUMERICAL SOLUTION

The size of reduction N of the ISVIE of the second kind will be chosen from considerations of practical convergence.

The integrals were calculated using the method of mechanical quadratures, in particular, the symmetric Gregory quadrature formula for equidistant nodes. The Cauchy problem for the differential equation (41) was solved by the Adams method (closed-type formulas) [1 – 5] of order m_1 with a local truncation error

$O(\Delta t^{m_1+1})$ [6 – 8, 10]. As a result of discretization, we obtain a system of linear algebraic equations (SLAE). Calculations have shown that with an increase in the reduction size N, the determinant of the SLAE matrix increases indefinitely. The SLAE is poorly defined: as the reduction size N tends to infinity, the value of the determinant of the SLAE matrix also tends to infinity. This is due to the fact that the kernels $Q_{11}(n,t)$, $Q_{22}(n,t)$ in (32), (33) have asymptotic $\exp(O(n))$ in the parameter n, $\tilde{Q}_{11}(n,t)$ and $\tilde{Q}_{22}(n,t)$ in (35) and (36) have asymptotic $O\left(1/n\right)\exp(O(n))$ in the parameter n. Methods of Tikhonov regularization and orthogonal polynomials do not work to neutralize such an exponential singularity. The approach [1 – 5] for solving problems of dynamics makes it impossible to study the impact of elastic shells of the S.P. Timoshenko type and elastic bodies on an elastic foundation [6 – 8, 10]. In addition, this approach makes it possible to determine the stress-strain state only on the surface of the medium into which the striker penetrates.

CONCLUSIONS

As a result of an attempt to solve the axisymmetric problem of the impact of a spherical fine shell of the S.P. Timoshenko type on the surface of an elastic half-space, applying the method of reduction of dynamic problems to infinite systems of Voltaire's equations of the second kind, the limitations of this technique were revealed. This technique does not allow solving plane and axisymmetric problems of dynamics for refined shells of the S.P. Timoshenko type and elastic bodies.

To solve [9, 11 – 14] the problems of impact and nonstationary interaction [15 – 19], the elastoplastic formulation [20 – 22] can be used. It should be noted that to calibrate the computational [1] process in the elastoplastic formulation at the elastic stage, it is convenient and expedient to use the technique [1 – 5] for solving the problems of dynamics, developed in [6 – 8, 10].

REFERENCES

1. **Bogdanov V.R., 2018.** Impact a circular cylinder with a flat on an elastic layer. Transfer of Innovative Technologies. Vol.1(2), 68-74.
2. **Bogdanov V.R., 2017.** Impact of a hard cylinder with flat surface on the elastic layer. Underwater Technologies, Vol.05, 8-15.
3. **Bogdanov V.R., Lewicki H.R., Pryhodko T.B., Radzivill O.Y., Samborska L.R., 2009.** The planar problem of the impact shell against elastic layer. Visnyk NTU, Kyiv, Nr. 18, 281-292 (in Ukrainian).
4. **Kubenko V.D., Bogdanov V.R., 1995.** Planar problem of the impact of a shell on an elastic half-space. International Applied Mechanics, 31, No. 6, 483-490.
5. **Kubenko V.D., Bogdanov V.R., 1995.** Axisymmetric impact of a shell on an elastic halfspace. International Applied Mechanics, 31, No. 10, 829-835.
6. **Kubenko V.D., Popov S.N., Bogdanov V.R., 1995.** The impact of elastic cylindrical shell with the surface of elastic half-space. Dop. NAN Ukrainy, No. 7, 40-44 (in Ukrainian).
7. **Kubenko V.D., Popov S.N., 1988.** Plane problem of the impact of hard blunt body on the surface of an elastic half-space. Pricl. Mechanika, 24, No. 7, 69-77 (in Russian).
8. **Popov S.N., 1989.** Vertical impact of the hard circular cylinder lateral surface on the elastic half-space. Pricl. Mechanika, 25, No. 12, 41-47 (in Russian).
9. **Bogdanov V.R., Sulim G.T., 2016.** Determination of the material fracture toughness by numerical analysis of 3D elastoplastic dynamic deformation. Mechanics of Solids, 51(2), 206-215; DOI 10.3103/S0025654416020084.
10. **Bogdanov V.R., 2015.** A plane problem of impact of hard cylinder with elastic layer. Bulletin of University of Kyiv, Mathematics. Mechanics, No. 34, 42-47 (in Ukrainian).
11. **Bogdanov V.R., Sulym G.T., 2013.** Plain deformation of elastoplastic material with profile shaped as a compact specimen (dynamic loading). Mechanics of Solids, May, 48(3), 329–336, DOI 10.3103/S0025654413030096.
12. **Bogdanov V.R., Sulym G.T., 2013.** A modeling of plastic deformation's growth under impact, based on a numerical solution of the plane stress deformation problem. Vestnik Moskovskogo Aviatsionnogo Instituta, Vol. 20, Issue 3, 196-201 (in Russian).
13. **Bogdanov V.R., 2009.** Three dimension problem of plastic deformations and stresses con-

centration near the top of crack. Bulletin of University of Kyiv, Series Physics & Mathematics, No. 2, 51-56 (in Ukrainian).
14. **Bogdanov V.R., Sulym G.T., 2012.** The plane strain state of the material with stationary crack with taking in account the process of unloading. Mathematical Methods and Physicomechanical Fields, Lviv, 55, No. 3, 132-138 (in Ukrainian).
15. **Bogdanov V.R., Sulym G.T., 2010.** The crack growing in compact specimen by plastic-elastic model of planar stress state. Bulletin of University of Kyiv, Series Physics & Mathematics, No. 4, 58-62 (in Ukrainian).
16. **Bogdanov V.R., Sulym G.T., 2010.** The crack clevage simulation based on the numerical modelling of the plane stress state. Bulletin of University of Lviv, Series Physics & Mathematics, No. 73, 192-204 (in Ukrainian).
17. **Bohdanov V.R., Sulym G.T., 2011.** Evaluation of crack resistance based on the numerical modelling of the plane strained state. Material Science, 46, No. 6, 723-732.
18. **Bogdanov V.R., Sulym G.T., 2011.** The clevage crack simulation based on the numerical modelling of the plane deformation state. Scientific collection Problems of Calculation Mechanics and Constructions Strength, Dnepropetrovsk, No. 15, 33-44 (in Ukrainian).
19. **Bogdanov V.R., Sulym G.T., 2010.** Destruction toughness determination based on the numerical modelling of the three dimension dynamic problem. International scientific collection Strength of Machines and Constructions, Kyiv, No. 43, 158-167 (in Ukrainian).
20. **Bogdanov V.R., Sulym G.T., 2012.** A three dimension simulation of process of growing crack based on the numerical solution. Scientific collection Problems of Calculation Mechanics and Constructions Strength, Dnepropetrovsk, No. 19, 10-19 (in Ukrainian).
21. **Bogdanov V.R., Sulym G.T., 2012.** The crack clevage simulation in a compact specimen based on the numerical modelling of the three dimension problem. Scientific collection Methods of Solving Applied Problems in Solid Mechanics, Dnepropetrovsk, No. 13, 60-68 (In Ukrainian).
22. **Bogdanov V.R., 2011.** About three dimension deformation of an elastic-plastic material with the profile of compact shape. Theoretical and Applied Mechanics, Donetsk, No. 3 (49), 51-58 (in Ukrainian).
23. **Fedotenkov G.V., 2001.** Cylinder shell impact along elastic semi-plane. Moscow Aviation Institute, 100 p. (In Russian).
24. **Mihailova E.Y., Tarlakovsky D.V., Vahterova Y.A., 2018.** generalized linear model of the dynamics of thin elastic shells. Scientific Notes of Kazan University, Series of Physics and Mathematics, 160(3), 561-577 (in Russian).
25. **Lokteva N.A., Serduk D.O., Skopintsev P.D., Fedotenkov G.V., 2020.** Non-stationary stress-deformed state of a composite cylindrical shell. Mechanics of Composite Materials and Structures, 26(4), 544-559; DOI: 10.33113/mkmk.ras.2020.26.04.544_559.08 (in Russian).
26. **Vestyak A.V., Igumnov L.A., Tarlakovskii D.V., Fedotenkov G.V., 2016.** The influence of non-stationary pressure on a thin spherical shell with an elastic filler. Computational Continuum Mechanics. 9(4), 443-452 (in Russian); DOI: 10.7242/1999-6691/2016.9.4.37.
27. **Afanasyeva O.A. Mikhailova E.Y. Fedotenkov G.V., 2012.** Random phase of contact interaction of a spherical shell and elastic half space. Problems of Computer Mechanics and Strength of Structures, 20, 19-26 (in Russian).
28. **Michailova E.Y., Tarlakovsky D.V., Fedotenkov G.V., 2014.** Non-stationary contact of a spherical shell and an elastic half-space. Works MAI. 78, 1-26 (in Russian).
29. **Igumnov L.A., Okonechnikov A.S., Tarlakovskii D.V., Fedotenkov G.V. 2013.** Plane nonsteady-state problem of motion of the surface load on an elastic half-space. Mathematical Methods and Physicomechanical Fields, Lviv, 56, No. 2, 157-163 (In Russian).
30. **Kuznetsova E.L., Tarlakovsky D.V., Fedotenkov G.V., Medvedsky A.L., 2013.** Influence of non-stationary distributed load on the surface of the elastic layer. Works MAI. 71, 1-21. (In Russian)
31. **Fedotenkov G.V., Tarlakovsky D.V., Vahterova Y.A., 2019.** Identification of Non-stationary Load Upon Timoshenko Beam. Lobachevskii Journal of Mathematics, 40(4), 439-447.
32. **Vahterova Y.A., Fedotenkov G.V., 2020.** The inverse problem of recovering an unsteady linear load for an elastic rod of finite length. Journal of Applied Engineering Science, 18(4), 687-692, DOI:10.5937/jaes0-28073.
33. **Gorshkov A.G., Tarlakovsky D.V., 1985.** Dynamic contact problems with moving boundaries. Nauka, Fizmatlit, 352 (in Russian).
34. **Sagomonian A.J., 1985.** Stress waves in a continuous medium. Moscow University Publishing House, 416 (in Russian).

Один подход к осесимметричной задаче удара оболочек типа С. П. Тимошенко об упругое полупространство

Владислав Богданов

Аннотация. Уточненная модель С.П. Тимошенко позволяет учесть вращение и инерцию такого вращения поперечного сечения оболочки. Возмущения распространяются в оболочках типа С.П. Тимошенко с конечной скоростью. Поэтому изучение динамики распространения волновых процессов в тонких оболочках типа С.П. Тимошенко является важным аспектом, так же как важно исследование волновых процессов удара в упругом основании, в которое проникает ударник. Хорошо изучены метод сведения решения задач динамики к решению бесконечной системы интегральных уравнений Вольтерра второго рода и сходимость этого решения. Такой подход успешно применялся для случаев исследования задач об ударе твердых тел и упругих тонких оболочек типа Кирхгофа – Лява об упругое полупространство и слой. В данной работе сделана попытка решения осесимметричной задачи об ударе упругой тонкой сферической оболочки типа С.П. Тимошенко об упругое полупространство методом сведения задач динамики к решению бесконечной системы интегральных уравнений Вольтерра второго рода. Показано, что такой подход неприемлем для исследуемой в данной статье осесимметричной задачи. Дискретизация с использованием методов Грегори для численного интегрирования и Адамса для решения задачи Коши для полученной бесконечной системы уравнений Вольтерра второго рода приводит к решению плохо определенной системы линейных алгебраических уравнений: при увеличении порядка редукции определитель такой системы стремиться к бесконечности. Данная методика не позволяет решать плоские и осесимметричные задачи динамики для тонких оболочек типа С. П. Тимошенко и упругих тел. Это показывает ограничения такого подхода и объясняет необходимость разработки других математических подходов и моделей. Следует отметить, что для калибровки вычислительного процесса в упругопластической постановке на упругой стадии удобно и целесообразно использовать технику сведения задач динамики для решения бесконечной системы интегральных уравнений Вольтерра второго рода.

Ключевые слова: удар, упругость, упругопластичность, полупространство, осесимметричная задача, тонкая сферическая оболочка, С.П. Тимошенко.