Problem of plane strain state of two-layer body in dynamic elastic-plastic formulation (Part III)

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Received 01.09.2022, accepted 20.12.2022
https://doi.org/10.32347/tit.2022.51.0302

Abstract. Composites materials are artificially created materials that consist of two or more components that differ in composition and are separated by a pronounced boundary. The development of modern composite materials is associated with the discovery of high-strength whiskers, with the study and use of aluminides and high-strength alloys. At present, various composite materials have been developed and used: fibrous; reinforced with whiskers and continuous crystals and fibres of refractory compounds and elements; dispersion-hardened materials; layered materials; alloys with directional crystallization of eutectic structures; alloys with intermetallic hardening. There are many technologies for producing composites: imbibition of reinforcing fibres with matrix (base) material; cold pressing of components followed by sintering; sediment of the matrix by plasma spraying on the hardener, followed by compression; batch diffusion welding of multilayer tapes of components; joint rolling of reinforcing elements with a matrix, and etc. The use of composites makes it possible to reduce the weight of aircraft, cars, ships, increase the efficiency of engines, and create new constructions with high performance and reliability. The development of composites with high impact resistance is an important direction in the industry. The strength characteristics of a layered composite material are decisive under shear loads, loading of the composite in directions other than the orientation of the layers, and cyclic loading. In this paper, we study the non-stationary interaction of an absolutely rigid body on a two-layer reinforced composite material. The action of the striker is replaced by a non-stationary vertical even distributed load, which changes according to a linear function, in the area of initial contact, which is assumed to be unchanged over time.

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In contrast to the previous articles (Parts I and II), in this papers there is an investigation of the strain-stress state, the fields of the Odquist parameter and normal stresses depending on the material of the first (upper) layer.

Keywords: plane, strain, impact, composite material, armed material, reinforced material, elastic-plastic, deformation.

INTRODUCTION

In order for the composite to have stable properties, it is necessary that chemical and mechanical compatibility be ensured. Chemical compatibility includes thermodynamic and kinetic compatibility. Thermodynamic compatibility is the ability of the matrix and reinforcing elements to be in a state of thermodynamic equilibrium for an unlimited time at the temperature of production and operation. A limited number of composite materials consisting of components that are practically insoluble in each other over a wide temperature range are thermodynamically compatible under isothermal conditions. Most composites consist of thermodynamically incompatible components, for which only possible phase equilibria and direction of reactions can be determined from phase diagrams. Kinetic compatibility is the ability of components to be in a state of metastable equilibrium, controlled by factors such as adsorption, diffusion rate, chemical reaction rate. Thermodynamically incompatible components of the composite in certain temperature-time intervals using new optimal technologies can be kinetically compatible and work quite reliably, Mechanical compatibility consists in matching the elastic constants, coefficients of thermal expansion and plasticity indices of the components, allowing to achieve bond strength for transferring stresses through the joint boundaries of materials. The materials steel/metal and glass have fairly good chemical and mechanical compatibility.

In [1, 2], a new approach to solving the problems of impact and nonstationary interaction in the elastoplastic mathematical formulation was developed. In these papers like in non-stationary problems [3], the action of the striker is replaced by a distributed load in the contact area, which changes according to a linear law. The contact area remains constant. The developed elastoplastic formulation makes it possible to solve impact problems when the dynamic change in the boundary of the contact area is considered and based on this the movement of the striker as a solid body with a change in the penetration speed is taken into account. Also, such an elastoplastic formulation makes it possible to consider the hardening of the material in the process of nonstationary and impact interaction.

The solution of problems for composite cylindrical shells [4], elastic layer [5], were developed using method of the influence functions.

In contrast from the work [6], in this paper, we investigate the impact process of hard body with plane area of its surface on the top of the composite beam which consists first thin metal layer and second main glass layer. In contrast from the works [7, 8], the fields of plastic deformations and, stresses were determined for different materials of the first top layer of composite base. First top layers made from aluminum, titan and steel were investigated.

PROBLEM FORMULATION AND SOLUTION ALGORITHM

Deformations and their increments [1 – 3], Odquist parameter, effective and principal stresses are obtained from the numerical solution of the dynamic elastic-plastic interaction problem of infinite composite beam $\{-L/2 \le x \le L/2;\ \ 0 \le y \le B; -\infty \le z \le \infty\}$ in the plane of its cross section in the form of rectangle as in [7, 8]. It is assumed that the stress-strain state in each cross section of the cylinder is the same, close to the plane deformation, and therefore it is necessary to solve the equation for only one section in the form of a rectangle $\Sigma = L \times B$ with two layers: first steel layer $\{-L/2 \le x \le L/2; -\infty \le z \le \infty;\ B-h \le y \le B\}$ and second glass layer $\{-L/2 \le x \le L/2;\ 0 \le y \le B-h; -\infty \le z \le \infty\}$ contacts absolute hard half-space $\{y \le 0\}$. We assume that the contact between the lower surface of the first metal layer and the upper surface of the second glass layer is ideally rigid.

From above on a body the absolutely rigid drummer contacting along a segment $\{|x| \le A;\ y = B\}$. Its action is replaced by an even distributed stress $-P$ in the contact region, which changes over time as a linear function $P = p_{01} + p_{02}t$. Given the symmetry of the deformation process relative to the line $x = 0$, only the right part of the cross section is considered below (Fig. 1).

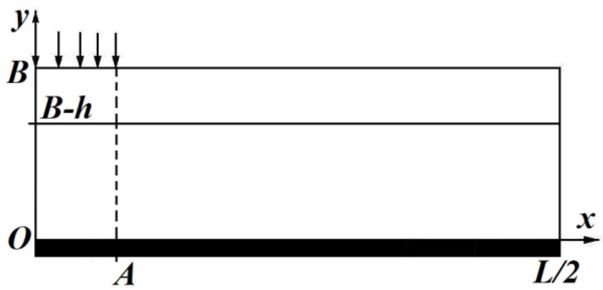

Fig. 1. Geometric scheme of the problem

The equations of the plane dynamic theory are considered, for which the components of the displacement vector $\mathbf{u} = (u_x, u_y)$ are related to the components of the strain tensor by Cauchy relations:

$$\varepsilon_{xx}=\frac{\partial u_x}{\partial x},\quad \varepsilon_{yy}=\frac{\partial u_y}{\partial y},\quad \varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right).$$

The equations of motion of the medium have the form:

$$\begin{aligned}&\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}=\rho\frac{\partial^2 u_x}{\partial t^2},\\&\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}=\rho\frac{\partial^2 u_y}{\partial t^2},\end{aligned}\quad (1)$$

where ρ – material density.

The boundary and initial conditions of the problem have the form:

$$\begin{aligned}&x=0,\ 0<y<B: u_x=0,\ \sigma_{xy}=0,\\&x=L/2, 0<y<B: \sigma_{xx}=0,\ \sigma_{xy}=0,\\&y=0,\ 0<x<L/2:\ u_y=0,\ \sigma_{xy}=0,\\&y=B,\ 0<x<A:\ \sigma_{yy}=-P,\ \sigma_{xy}=0,\\&y=B,\ A<x<L/2:\ \sigma_{yy}=0,\ \sigma_{xy}=0.\end{aligned}\quad (2)$$

$$\begin{aligned}&u_x\big|_{t=0}=0,\ u_y\big|_{t=0}=0,\ u_z\big|_{t=0}=0,\\&\dot{u}_x\big|_{t=0}=0,\ \dot{u}_y\big|_{t=0}=0,\ \dot{u}_z\big|_{t=0}=0.\end{aligned}\quad (3)$$

The determinant relations of the mechanical model are based on the theory of non-isothermal plastic flow of the medium with hardening under the condition of Huber-Mises fluidity. The effects of creep and thermal expansion are neglected. Then, considering the components of the strain tensor by the sum of its elastic and plastic components [9, 10], we obtain expression for them:

$$\begin{aligned}&\varepsilon_{ij}=\varepsilon_{ij}^e+\varepsilon_{ij}^p,\ d\varepsilon_{ij}^p=s_{ij}d\lambda,\\&\varepsilon_{ij}^e=\frac{1}{2G}s_{ij}+K\sigma+\varphi.\end{aligned}\quad (4)$$

here $s_{ij}=\sigma_{ij}-\delta_{ij}\sigma$ – stress tensor deviator; δ_{ij} – Kronecker symbol; E – modulus of elasticity (Young's modulus); G – shear modulus; $K_1=(1-2\nu)/(3E)$, $K=3K_1$ – volumetric compression modulus, which binds in the ratio $\varepsilon=K\sigma+\phi$ volumetric expansion 3ε (thermal expansion $\phi\equiv 0$); $\sigma=(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})/3$ – mean stress; $d\lambda$ – some scalar function [11], which is determined by the shape of the load surface and we assume that this scalar function is quadratic function of the stress deviator s_{ij} [9 – 11].

$$d\lambda=\begin{cases}0 \quad (f\equiv\sigma_i^2-\sigma_S^2(T)<0)\\ \dfrac{3d\varepsilon_i^p}{2\sigma_i} \quad (f=0, df=0)\\ (f>0-\text{inadmissible})\end{cases},\quad (5)$$

$$d\varepsilon_i^p=\frac{\sqrt{2}}{3}\left(\left(d\varepsilon_{xx}^p-d\varepsilon_{yy}^p\right)^2+\left(d\varepsilon_{xx}^p-d\varepsilon_{zz}^p\right)^2+\left(d\varepsilon_{yy}^p-d\varepsilon_{zz}^p\right)^2+6\left(d\varepsilon_{xy}^p\right)^2\right)^{1/2},$$

$$\sigma_i=\frac{1}{\sqrt{2}}\left(\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{xx}-\sigma_{zz}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+6\sigma_{xy}^2\right)^{1/2}.$$

The material is strengthened with a hardening factor η^* [1 – 3]:

$$\begin{aligned}&\sigma_S(T)=\sigma_{02}(T_0)\left(1+\frac{\kappa(T)}{\varepsilon_0}\right)^{\eta^*},\\&\varepsilon_0=\frac{\sigma_{02}(T_0)}{E},\end{aligned}\quad (6)$$

where T – temperature; κ – Odquist parameter, $T_0=20^\circ C$, η^* – hardening coefficient; $\sigma_S(T)$ – yield strength after hardening of the material at temperature T.

Rewrite (4) in expanded form:

$$\begin{aligned}&d\varepsilon_{xx}=d\left(\frac{\sigma_{xx}-\sigma}{2G}+K\sigma\right)+(\sigma_{xx}-\sigma)d\lambda,\\&d\varepsilon_{yy}=d\left(\frac{\sigma_{yy}-\sigma}{2G}+K\sigma\right)+(\sigma_{yy}-\sigma)d\lambda,\\&d\varepsilon_{zz}=d\left(\frac{\sigma_{zz}-\sigma}{2G}+K\sigma\right)+(\sigma_{zz}-\sigma)d\lambda,\\&d\varepsilon_{xy}=d\left(\frac{\sigma_{xy}}{2G}\right)+\sigma_{xy}d\lambda,\end{aligned}\quad (7)$$

In contrast to the traditional plane deformation, when $\Delta\varepsilon_{zz}(x,y)=\text{const}$, for a refined description of the deformation of the specimen, taking into account the possible increase in longitudinal elongation $\Delta\varepsilon_{zz}$, we present in its form [2, 3, 12]:

$$\Delta\varepsilon_{zz}(x,y)=\Delta\varepsilon_{zz}^{0}+\Delta\chi_x x+\Delta\chi_y y, \quad (8)$$

where unknown $\Delta\chi_x$ and $\Delta\chi_y$ describe the bending of the prismatic body (which simulates the plane strain state in the solid mechanics) in the *Ozx* and *Ozy* planes, respectively, and $\Delta\varepsilon_{zz}^{0}$ – the increments according to the detected deformation bending along the fibers $x=y=0$.

The solution algorithm is the same as in [7, 8].

NUMERICAL SOLUTION

The explicit scheme of the finite difference method was used with a variable partitioning step along the axes *Ox* (*M* elements) and *Oy* (*N* elements). The step between the split points was the smallest in the area of the layers contact and at the boundaries of the computational domain. Since the interaction process is fleeting, this did not affect the accuracy in the first thin layer, areas near the boundaries, and the adequacy of the contact interaction modelling.

The use of finite differences [13] with variable partition step for wave equations is justified in [14], and the accuracy of calculations with an error of no more than $O\left((\Delta x)^2+(\Delta y)^2+(\Delta t)^2\right)$ where Δx, Δy and Δt – increments of variables: spatial *x* and *y* and time *t*. A low rate of change in the size of the steps of the partition mesh was ensured. The time step was constant.

The resolving system of linear algebraic equations with a banded symmetric matrix was solved by the Gauss method according to the Cholesky scheme.

In [15], during experiments, compact samples were destroyed in 21 – 23 ms. The process of destruction of compact specimens from a material of size and with contact loading as in [15] was modelled in a dynamic elastoplastic formulation, considering the unloading of the material and the growth of a crack according to the local criterion of brittle fracture. The samples were destroyed in 23 ms. This confirms the correctness and adequacy of the developed formulation and model.

Figs. 2 – 29 show the results of calculations of two layers specimens with a hardening factor of the material $\eta^*=0{,}05$. The first high layer has made from hard steel and its thickness was equal $h=0.5$ mm. The second main low layer has made from quartz glass. Contact between two layers is an ideal. Calculations were made at the following parameter values: temperature $T=50\ ^{\circ}\text{C}$; the size of the contact zone $a=2A=0.5$ mm; $L=60$ mm; $B=10$ mm; $\Delta t=3.21\cdot10^{-8}$ s; $p_{01}=8$ MPa; $p_{02}=10$ MPa; $M=62$; $N=100$. The smallest splitting step was 0,005 mm, and the largest 2,6 mm ($\Delta x_{\min}=0{,}005$ mm; $\Delta y_{\min}=0{,}01$ mm (only the first layer); $\Delta x_{\max}=2{,}6$ mm; $\Delta y_{\max}=0{,}65$ mm).

The fields of the Odquist parameter κ, normal stresses σ_{xx} and σ_{yy} respectively to aluminium shown at the Figs. 2, 5, 8, 11, 14, 17, 20, 23, 26; respectively to titan shown at the Figs. 3, 6, 9, 12, 15, 18, 21, 24, 27; respectively to steel shown at the Figs. 4, 7, 10, 13, 16, 19, 22, 25, 28.

Figs. 2 – 4, 11 – 13, 20 – 22 show the fields of the Odquist parameter κ, normal stresses σ_{xx} and σ_{yy} respectively at the time $t_1=2.57\cdot10^{-6}$ s. The fields of Odquist parameter κ, normal stresses σ_{xx} and σ_{yy} respectively are at the Figs. 5 – 7, 14 – 16, 23 – 25 at the time $t_2=3.05\cdot10^{-6}$s. Figs. 8 – 10, 17 – 19, 26 – 28 show the fields of κ, σ_{xx} and σ_{yy} respectively at the time $t_3=3.53\cdot10^{-6}$s.

Below, all the conclusions are made on the basis of the studied hard steel and two metals: aluminium and titanium. From Figs. 2 – 4, 11 – 13, 20 – 22 it can be seen that at the beginning of the nonlinear process of non-stationary interaction, the zone of plastic deformations is larger

Fig. 2. Odquist parameter κ when $t = t_1$

Fig. 3. Odquist parameter κ when $t = t_1$

Fig. 4. Odquist parameter κ when $t = t_1$

Fig. 5. Odquist parameter κ when $t = t_2$

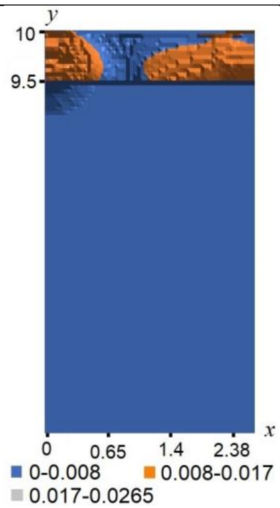

Fig. 6. Odquist parameter κ when $t = t_2$

Fig. 7. Odquist parameter κ when $t = t_2$

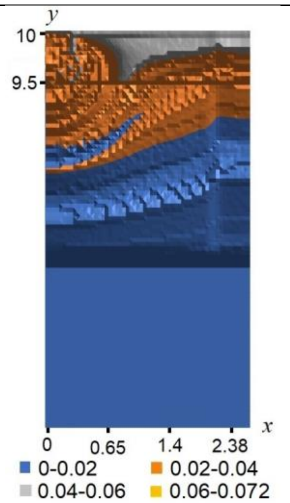

Fig. 8. Odquist parameter κ when $t = t_3$

Fig. 9. Odquist parameter κ $t = t_3$

Fig. 10. Odquist parameter κ $t = t_3$

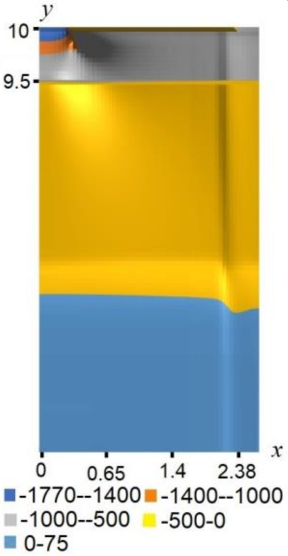

Fig. 11. Stress σ_{xx} when $t = t_1$

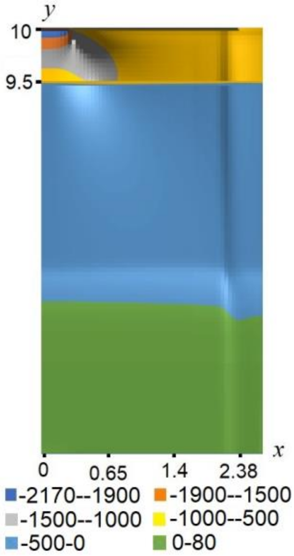

Fig. 12. Stress σ_{xx} when $t = t_1$

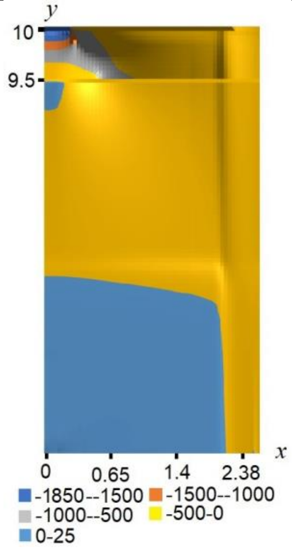

Fig. 13. Stress σ_{xx} when $t = t_1$

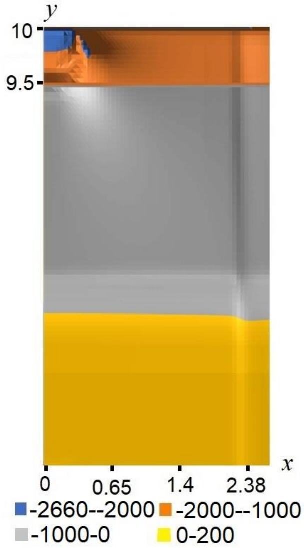

Fig. 14. Stress σ_{xx} when $t = t_2$

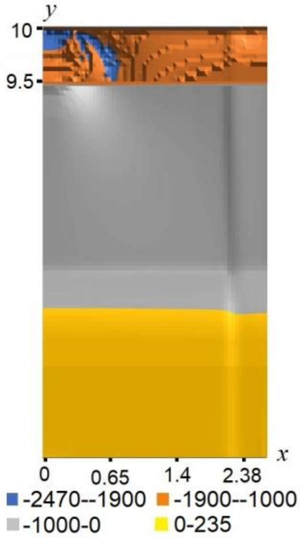

Fig. 15. Stress σ_{xx} when $t = t_2$

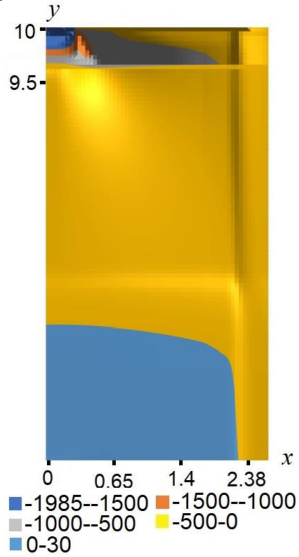

Fig. 16. Stress σ_{xx} when $t = t_2$

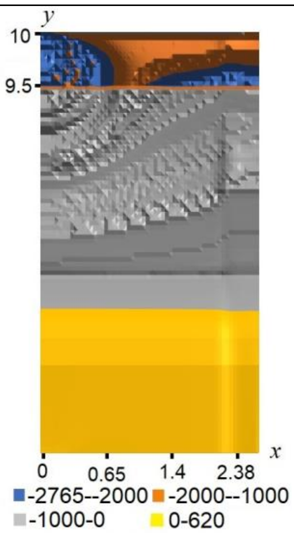

Fig. 17. Stress σ_{xx} when $t = t_3$

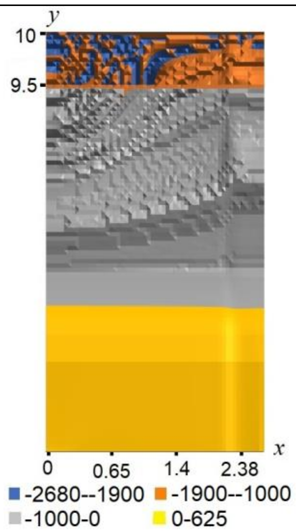

Fig. 18. Stress σ_{xx} when $t = t_3$

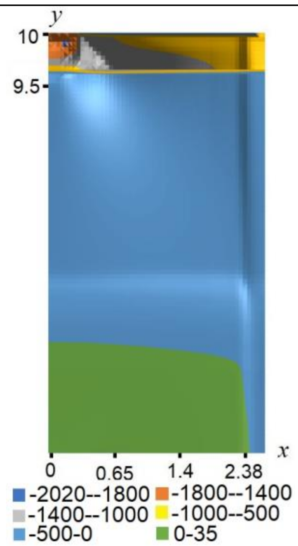

Fig. 19. Stress σ_{xx} when $t = t_3$

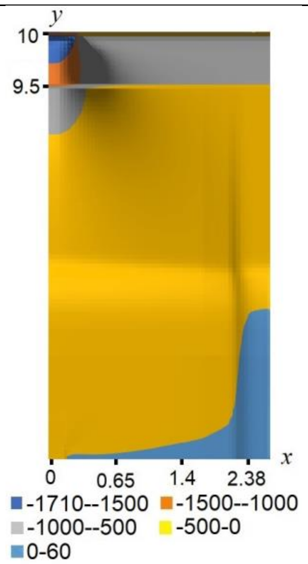

Fig. 20. Stress σ_{yy} when $t = t_1$

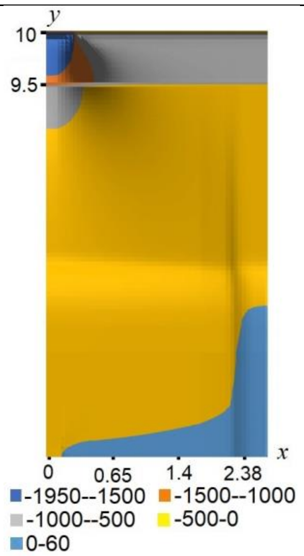

Fig. 21. Stress σ_{yy} when $t = t_1$

Fig. 22. Stress σ_{yy} when $t = t_1$

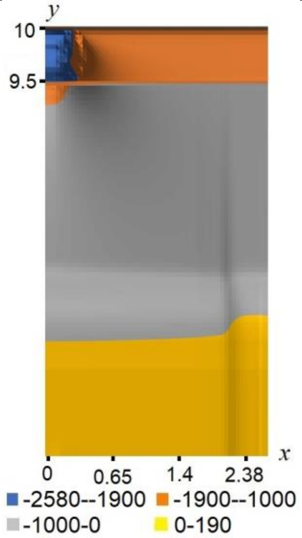

Fig. 23. Stress σ_{yy} when $t = t_2$

Fig. 24. Stress σ_{yy} when $t = t_2$

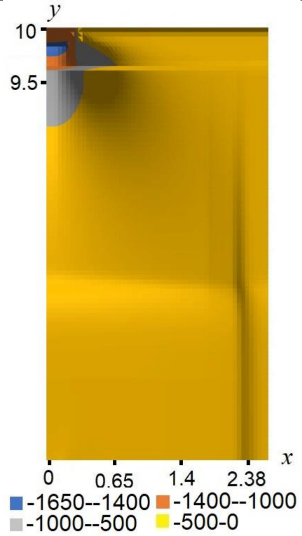

Fig. 25. Stress σ_{yy} when $t = t_2$

Fig. 26. Stress σ_{yy} when $t = t_3$

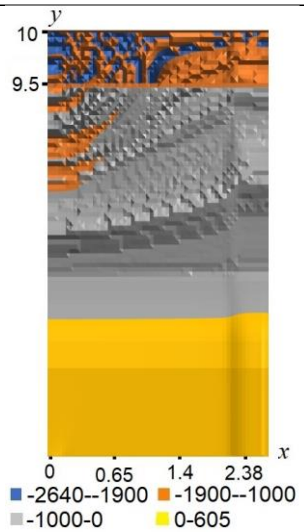

Fig. 27. Stress σ_{yy} when $t = t_3$

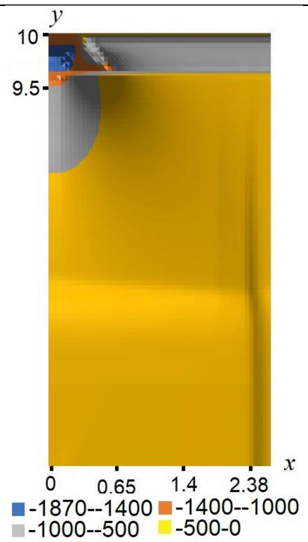

Fig. 28. Stress σ_{yy} when $t = t_3$

when the metal of top layer is softer. In this case, the largest plastic deformations, normal stresses σ_{xx} and σ_{yy} occur in titanium. At the moment of time $t = t_2$, in the case of the titanium top layer, the area of summary plastic deformation is the largest and covers the entire top layer, the smallest area of plastic deformation in the steel layer. The largest plastic deformations occur precisely in titanium, and the smallest in steel. The largest normal stresses occur in the aluminium material. As can be seen from Figs. 8 – 10, 17 – 19, 26 – 28 at the moment of time $t = t_3$ the smallest area of summary plastic deformations occurs in the steel layer, so in steel this area is directly under the contact zone of the impactor with the upper surface. The value of plastic deformation in steel is 6 times less than in titanium. The maximum normal stresses occur in aluminium as well as at the moment of time $t = t_2$. Based on the analysis of the obtained results, it can be concluded that in the case of soft, more plastic materials, the absolute values of occurred normal stresses and summary plastic deformation are larger. It seems reasonable to produce a four-layer composite reinforced material, in which the upper thin layer will be steel, the third layer from the top will be aluminium, and between them there are two layers of glass. However, this is a different problem that requires numerical simulation and research in further investigations.

CONCLUSIONS

The developed methodology of solving dynamic contact problems in an elastic-plastic dynamic mathematical formulation makes it possible to model the processes of impact, shock and non-stationary contact interaction with the elastic composite base more adequately. In this work, the process of impact on a two-layer base, consisting of an upper thin layer of metal and a lower main layer of glass, is adequately modelled. The fields of summary plastic deformations and normal stresses arising in the base are calculated depending on the material of top layer of the composite base. The results obtained make it possible to design new composite reinforced armed materials.

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Задача о плоском деформированном состоянии двухслойного тела в динамической упругопластической постановке (Часть III)

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Аннотация. Композиционные материалы это искусственно созданные материалы, которые состоят из двух или более компонентов, различающихся по составу и разделенных выраженной границей. Развитие современных композиционных материалов непосредственно связано с открытием высокопрочных нитевидных кристаллов, разработкой новых армирующих материалов, с исследованиями и применением алюминидов и высокопрочных сплавов. В настоящее время разработаны и применяются различные композиционные материалы: волокнистые, армированные нитевидными кристалами и непрерывными кристалами и непрерывными волокнами тугоплавких соединений и элементов, дисперсно-упрочненные материалы, слоистые материалы, сплавы с направленной кристализацией эвтектических структур, сплавы с интерметаллидным упрочнением. Существует множество технологий получения композитов: пропитка армирующих волокон матричным (основным) материалом; холодное прессование компонентов с последующим спеканием; осаждение матрицы плазменным напылением на упрочнитель с последующим обжатием; пакетная диффузионная сварка многослойных лент компонентов; совместная прокатка армирующих элементов с матрицей и др. Применение композитов позволяет снизить массу летательных аппаратов, автомобилей, судов, увеличить Коэффициент полезного действия двигателей, создать новые конструкции, обладающие высокой работоспособностью и надежностью. Разработка композитов с высокой сопротивляемостью ударным нагрузкам является важным направлением в промышленности. Прочностные характеристики слоистого композиционного материала являются определяющими при сдвиговых нагрузках, нагружении композита в направлениях, отличных от ориентации слоев, и циклических нагружениях. В данной работе исследуется нестационарное взаимодействие абсолютно твердого тела на двухслойный упрочненный композитный материал. Действие ударника заменяется нестационарной вертикальной равномерно распределенной нагрузки, изменяющейся по линейному закону, в области начального контакта, который предполагается неизменным с течением времени. Предполагается жесткое сцепление слоев между собой. Процесс удара моделировался как нестационарная задача с равномерно распределенной нагрузкой в области контакта, изменяющейся по линейному закону. В отличие от предыдущих статей (часть I и II) в данной статье исследуется напряженно-деформированное состояние, поля параметра Одквиста и нормальных напряжений в зависимости от толщины первого (верхнего) слоя.

Ключевые слова: плоская деформация, удар, композитные материалы, армированные материалы, бронированные материалы, упругопластическая, деформация.