Problem of plane stress state of two-layer body in dynamic elastic-plastic formulation

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Abstract. Composite materials are widely used in industry and everyday life. Many different methods are used to calculate and develop composite materials. Many methods of calculation and design of such materials are successfully used. In this article, for the design of composite and reinforced materials, a technique for solving dynamic contact problems in more precise an elastic-plastic mathematical formulation is used. To consider the physical nonlinearity of the deformation process, the method of successive approximations is used, which makes it possible to reduce the nonlinear problem to a solution of the sequences of linear problems. The problem of a plane stress state (PStS) of a beam made from the composite reinforced double-layered material is being solved in dynamic elastic-plastic mathematical model. The reinforced or armed material consists of two layers: the upper (first) thin layer of solid steel and the lower (second) main layer of glass. This composite base is rigidly attached to an absolutely hard half-space. Rigid adhesion of the layers to each other is assumed. Glass is a very strong and very fragile material at the same time. The fragility of glass is due to the fact that there are many microcracks on the surface, and when a load is applied to the glass surface, these microcracks begin to grow and lead to the destruction of glass products. If we glue or immobilize the tops of microcracks on the surface, we will get a strong reinforced armed material that will be lighter, stronger and not subject to degradation of material properties such as aging, corrosion and creep. The impact process was modelled as a non-stationary plane stress state problem with an even distributed load in the contact area, which changes according to a linear law. The fields of the Odquist parameter and normal stresses were studied and compared to corresponding results of plane strain (PSS) problem with the same material of layers, same their thickness and size of the contact area. The upper reinforcing layer of metal or steel can be applied to the glass surface so that metal or steel atoms penetrate deeply, fill microcracks and bind their tops. The top layer can be quite thin.

Keywords: Plane, stress, impact, composite material, armed material, reinforced material, elastic-plastic, deformation.

INTRODUCTION

In [1 – 4], a new approach to solving the plane stress problems of impact and nonstationary interaction in the elastoplastic mathematical formulation was developed. In this papers like in non-stationary problems [5 – 7], the action of the striker is replaced by a distributed load in the contact area, which changes according to a linear law. The contact area remains constant. The developed elastoplastic formulation makes it possible to solve impact problems when the dynamic change in the boundary of the contact area is considered and based on this the movement of the striker as a solid body with a change in the penetration speed is taken into account. Also, such an elastoplastic formulation makes it possible to consider the hardening of the material in the process of nonstationary and impact interaction.

The solution of problems for composite cylindrical shells [8], elastic half-space [9], elastic layer [10], elastic rod [11, 12] were developed using method of the influence functions [13].

In contrast from the work [14], in this paper, we investigate the impact process of hard body with plane area of its surface on the top of the composite beam which consists first thin metal layer and second main glass layer. In contrast from the works [5 – 7], in this paper, the impact process of hard body with plane area of its surface on the top of the composite beam which consists first thin metal layer and second main glass layer was investigated as plane problem of stress state in elastic-plastic mathematical model. The fields of plastic deformations and, stresses were determined relative to the size of the area of initial contact.

PROBLEM FORMULATION

Deformations and their increments [15, 16], Odquist parameter, effective and principal stresses are obtained from the numerical solution of the dynamic elastic-plastic interaction problem of infinite composite beam $\{-L/2 \le x \le L/2;\ \ 0 \le y \le B; -\infty \le z \le \infty\}$ in the plane of its cross section in the form of rectangle. It is assumed that the stress-strain state in each cross section of the cylinder is the same, close to the plane deformation, and therefore it is necessary to solve the equation for only one section in the form of a rectangle $\Sigma = L \times B$ with two layers: first steel layer $\{-L/2 \le x \le L/2; -\infty \le z \le \infty;\ B-h \le y \le B\}$ and second glass layer $\{-L/2 \le x \le L/2;$ $0 \le y \le B-h; -\infty \le z \le \infty\}$ contacts absolute hard half-space $\{y \le 0\}$. We assume that the contact between the lower surface of the first metal layer and the upper surface of the second glass layer is ideally rigid.

From above on a body the absolutely rigid drummer contacting along a segment $\{|x| \le A;$ $y = B\}$. Its action is replaced by an even distributed stress in the contact region, which changes over time as a linear function $P = p_{01} + p_{02}t$. Given the symmetry of the deformation process relative to the line $x = 0$, only the right part of the cross section is considered below (Fig. 1). The calculations use known methods for studying the quasi-static elastic-plastic [16, 17 – 19] model, considering the non-stationarity of the load and using numerical integration implemented in the calculation of the dynamic elastic model [1 – 4].

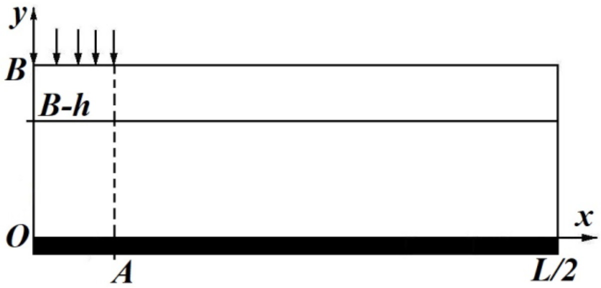

Fig. 1. Geometric scheme of the problem

The equations of the plane dynamic theory are considered, for which the components of the displacement vector $\mathbf{u} = (u_x, u_y)$ are related to the components of the strain tensor by Cauchy relations:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x},\quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y},\quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right).$$

The equations of motion of the medium have the form:

$$\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} &= \rho \frac{\partial^2 u_x}{\partial t^2}, \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} &= \rho \frac{\partial^2 u_y}{\partial t^2}, \end{aligned} \qquad (1)$$

where ρ – material density.

The boundary and initial conditions of the problem have the form:

$$
\begin{aligned}
&x=0,\ 0<y<B: u_x=0,\ \sigma_{xy}=0,\\
&x=L/2, 0<y<B: \sigma_{xx}=0,\ \sigma_{xy}=0,\\
&y=0,\ 0<x<L/2:\ u_y=0,\ \sigma_{xy}=0,\\
&y=B,\ 0<x<A:\ \sigma_{yy}=-P,\ \sigma_{xy}=0,\\
&y=B,\ A<x<L/2:\ \sigma_{yy}=0,\ \sigma_{xy}=0.
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
&u_x\big|_{t=0}=0,\ u_y\big|_{t=0}=0,\ u_z\big|_{t=0}=0,\\
&\dot{u}_x\big|_{t=0}=0,\ \dot{u}_y\big|_{t=0}=0,\ \dot{u}_z\big|_{t=0}=0.
\end{aligned} \tag{3}
$$

The determinant relations of the mechanical model are based on the theory of non-isothermal plastic flow of the medium with hardening under the condition of Huber-Mises fluidity. The effects of creep and thermal expansion are neglected. Then, considering the components of the strain tensor by the sum of its elastic and plastic components [19, 20], we obtain expression for them:

$$
\begin{aligned}
&\varepsilon_{ij}=\varepsilon_{ij}^{e}+\varepsilon_{ij}^{p},\ d\varepsilon_{ij}^{p}=s_{ij}d\lambda,\\
&\varepsilon_{ij}^{e}=\frac{1}{2G}s_{ij}+K\sigma+\varphi.
\end{aligned} \tag{4}
$$

here $s_{ij}=\sigma_{ij}-\delta_{ij}\sigma$ – stress tensor deviator; δ_{ij} – Kronecker symbol; E – modulus of elasticity (Young's modulus); G – shear modulus; $K_1=(1-2\nu)/(3E)$, $K=3K_1$ – volumetric compression modulus, which binds in the ratio $\varepsilon=K\sigma+\phi$ volumetric expansion 3ε (thermal expansion $\phi\equiv 0$); σ – mean stress; $d\lambda$ – some scalar function [16], which is determined by the shape of the load surface and we assume that this scalar function is quadratic function of the stress deviator s_{ij} [19, 20].

$$
d\lambda=\begin{cases}0 \quad (f\equiv\sigma_i^2-\sigma_S^2(T)<0)\\ \dfrac{3d\varepsilon_i^p}{2\sigma_i} \quad (f=0, df=0)\\ (f>0-\text{inadmissible})\end{cases}, \tag{5}
$$

$$
d\varepsilon_i^p=\frac{\sqrt{2}}{3}\left(\left(d\varepsilon_{xx}^p-d\varepsilon_{yy}^p\right)^2+\left(d\varepsilon_{xx}^p-d\varepsilon_{zz}^p\right)^2+\right.
$$

$$
\left.+\left(d\varepsilon_{yy}^p-d\varepsilon_{zz}^p\right)^2+6\left(d\varepsilon_{xy}^p\right)^2\right)^{1/2},
$$

$$
\sigma_i=\frac{1}{\sqrt{2}}\left(\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{xx}\right)^2+\right.
$$

$$
\left.+\left(\sigma_{yy}\right)^2+6\sigma_{xy}^2\right)^{1/2}.
$$

The material is strengthened with a hardening factor η^* [1, 2, 16 – 18]:

$$
\begin{aligned}
&\sigma_S(T)=\sigma_{02}(T_0)\left(1+\frac{\kappa(T)}{\varepsilon_0}\right)^{\eta^*},\\
&\varepsilon_0=\frac{\sigma_{02}(T_0)}{E},
\end{aligned} \tag{6}
$$

where T – temperature; κ – Odquist parameter, $T_0=20^\circ C$, η^* – hardening coefficient; $\sigma_S(T)$ – yield strength after hardening of the material at temperature T.

Rewrite (4) in expanded form:

$$
\begin{aligned}
&d\varepsilon_{xx}=d\left(\frac{\sigma_{xx}-\sigma}{2G}+K\sigma\right)+(\sigma_{xx}-\sigma)d\lambda,\\
&d\varepsilon_{yy}=d\left(\frac{\sigma_{yy}-\sigma}{2G}+K\sigma\right)+(\sigma_{yy}-\sigma)d\lambda,\\
&d\varepsilon_{xy}=d\left(\frac{\sigma_{xy}}{2G}\right)+\sigma_{xy}d\lambda.
\end{aligned} \tag{7}
$$

SOLUTION ALGORITHM

Let the nonstationary interaction [1 – 3, 15, 16] occur in a time interval $t\in[0, t_*]$. Then for every moment of time t:

$$\varepsilon_{xx}^{e}=\frac{\sigma_{xx}-\sigma}{2G}+K\sigma,\ \varepsilon_{yy}^{e}=\frac{\sigma_{yy}-\sigma}{2G}+K\sigma,$$
$$\varepsilon_{xy}^{e}=\frac{\sigma_{xy}}{2G},\ \varepsilon_{zz}^{e}=-\frac{\nu}{1-\nu}(\varepsilon_{xx}^{e}+\varepsilon_{yy}^{e}),$$
$$\frac{d\varepsilon_{xx}^{p}}{dt}=\left(\sigma_{xx}-\sigma\right)\frac{d\lambda}{dt},\ \frac{d\varepsilon_{yy}^{p}}{dt}=\left(\sigma_{yy}-\sigma\right)\frac{d\lambda}{dt},\tag{9}$$
$$\frac{d\varepsilon_{xy}^{p}}{dt}=\sigma_{xy}\frac{d\lambda}{dt},\ \varepsilon_{zz}^{p}=-\varepsilon_{xx}^{p}-\varepsilon_{yy}^{p}.$$

For numerical integration over time, Gregory's quadrature formula [21] of order $m_1=3$ with coefficients D_n was used. After discretisation in time with nodes $t_k=k\Delta t\in[0,t_*]$ $(k=0,K)$ for each value k we write down the corresponding node values of deformation increments:

$$\Delta\varepsilon_{xx,k}=B_1\sigma_{xx,k}+B_2\sigma_{yy,k}-b_{xx},$$
$$\Delta\varepsilon_{yy,k}=B_2\sigma_{xx,k}+B_1\sigma_{yy,k}-b_{yy},$$
$$\Delta\varepsilon_{xy,k}=B_3\sigma_{xy,k}-b_{xy},\tag{10}$$
$$\Delta\varepsilon_{zz,k}=B_4(\sigma_{xx,k}+\sigma_{yy,k})-$$
$$-\Delta\varepsilon_{xx,k}-\Delta\varepsilon_{yy,k}-b_{zz},$$

$$B_1=\frac{1}{3}\left(K+\frac{1}{G}+2D_0\Delta\lambda_k\right),$$
$$B_2=\frac{1}{3}\left(K-\frac{1}{2G}-D_0\Delta\lambda_k\right),$$
$$B_3=\frac{1}{2G}+D_0\Delta\lambda_k,$$
$$B_4=\frac{1-2\nu}{3(1-\nu)}\left(2K+\frac{1}{2G}\right),$$
$$b_{zz}=B_4(\sigma_{xx,k-1}+\sigma_{yy,k-1}),$$
$$b_{ij}=\frac{1}{2G}\sigma_{ij,k-1}+\delta_{ij}\left(K-\frac{1}{2G}\right)\sigma_{k-1}-$$
$$-\sum_{n=1}^{m_1-1}D_n\left(\sigma_{ij,k-n}-\delta_{ij}\sigma_{k-n}\right)\Delta\lambda_{k-n}\quad(i,j\,□\,x,y).$$

The solution of the system (10) gives expressions for the components of the stress tensor at each step:

$$\sigma_{xx,k}=A_1\Delta\varepsilon_{xx,k}+A_2\Delta\varepsilon_{yy,k}+Y_{xx},$$
$$\sigma_{yy,k}=A_2\Delta\varepsilon_{xx,k}+A_1\Delta\varepsilon_{yy,k}+Y_{yy},$$
$$\sigma_{xy,k}=A_3\Delta\varepsilon_{xy,k}+Y_{xy},$$
$$Y_{xx}=A_1b_{xx}+A_2b_{yy},$$
$$Y_{yy}=A_2b_{xx}+A_1b_{yy},\tag{11}$$
$$A_1=B_1\Big/\left(B_1^2-B_2^2\right),$$
$$A_2=-B_2\Big/\left(B_1^2-B_2^2\right),$$
$$Y_{xy}=A_3b_{xy},\ A_3=1/B_3.$$

Function $\psi=1/(2G)+\Delta\lambda$, which is characterizing the yield condition, taking into account (8), (9), (11) is:

$$\psi=\begin{cases}\dfrac{1}{2G}\quad(f<0)\\ \dfrac{1}{2G}+\dfrac{3\Delta\varepsilon_i^p}{2\sigma_i}\quad(f=0,\,df=0),\\ (f>0-\text{inadmissible})\end{cases}\tag{12}$$

$$\Delta\varepsilon_i^p=\frac{\sqrt{2}}{3}\left(\left(\Delta\varepsilon_{xx}^p-\Delta\varepsilon_{yy}^p\right)^2+\left(\Delta\varepsilon_{xx}^p-\Delta\varepsilon_{zz}^p\right)^2+\right.$$
$$\left.+\left(\Delta\varepsilon_{yy}^p-\Delta\varepsilon_{zz}^p\right)^2+6\left(\Delta\varepsilon_{xy}^p\right)^2\right)^{1/2},$$
$$\Delta\varepsilon_{xx}^p=\Delta\varepsilon_{xx}-\Delta\varepsilon_{xx}^e,\ \Delta\varepsilon_{yy}^p=\Delta\varepsilon_{yy}-\Delta\varepsilon_{yy}^e,$$
$$\Delta\varepsilon_{xy}^p=\Delta\varepsilon_{xy}-\Delta\varepsilon_{xy}^e,\ \Delta\varepsilon_{zz}^p=-\Delta\varepsilon_{xx}^p-\Delta\varepsilon_{yy}^p,$$
$$\varepsilon_{xx}^e=\frac{1}{2G}\sigma_{xx}+\left(K-\frac{1}{2G}\right)\sigma,$$
$$\varepsilon_{yy}^e=\frac{1}{2G}\sigma_{yy}+\left(K-\frac{1}{2G}\right)\sigma,$$
$$\varepsilon_{xy}^e=\frac{1}{2G}\sigma_{xy},\ \sigma=(\sigma_{xx}+\sigma_{yy})/3.$$

To take into account [15, 16] the physical nonlinearity contained in conditions (12), the method of successive approximations is used, which makes it possible to reduce a nonlinear problem to a sequence of linear problems [16 – 18]:

$$\psi^{(n+1)} = \begin{cases} \psi^{(n)} p + \dfrac{1-p}{2G}, & \text{if } \ \sigma_{iS} < -Q, \\ \psi^{(n)}, & \text{if } \ -Q < \sigma_{iS} < Q, \\ \psi^{(n)} \dfrac{\sigma_i^{(n)}}{\sigma_S(T)}, & \text{if } \ \sigma_{iS} > Q, \end{cases}$$

$$\sigma_{iS} = \sigma_i^{(n)} - \sigma_S(T), \qquad (13)$$

where Q – the value of the largest deviation of the stress intensity $\sigma_i^{(n)}$ in step n from the strengthened yield strength; n – is the approximation number

The stresses and strains used above were determined for each unit cell from the numerical solution at each point in time $t_k = k\Delta t$.

NUMERICAL SOLUTION

The explicit scheme of the finite difference method was used with a variable partitioning step along the axes *Ox* (*M* elements) and *Oy* (*N* elements). The step between the split points was the smallest in the area of the layers contact and at the boundaries of the computational domain. Since the interaction process is fleeting, this did not affect the accuracy in the first thin layer, areas near the boundaries, and the adequacy of the contact interaction modelling.

The use of finite differences [21] with variable partition step for wave equations is justified in [22], and the accuracy of calculations with an error of no more than $O\left((\Delta x)^2 + (\Delta y)^2 + (\Delta t)^2\right)$ where Δx, Δy and Δt – increments of variables: spatial x and y and time t. A low rate of change in the size of the steps of the partition mesh was ensured. The time step was constant.

The resolving system of linear algebraic equations with a banded symmetric matrix was solved by the Gauss method according to the Cholesky scheme.

Figs. 2 – 19 show the results of calculations of two layers specimens with a hardening factor of the material $\eta^* = 0,05$. The first high layer has made from hard steel. The second main low layer has made from quartz glass. Contact between two layers is an ideal. Calculations were made at the following parameter values: temperature $T = 50\ ^\circ\text{C}$; $L = 60$ mm; $B = 10$ mm; $h = 0.3$ mm; $\Delta t = 3.21 \cdot 10^{-8}$ s; $p_{01} = 8$ MPa; $p_{02} = 10$ MPa; $M = 62$; $N = 100$. The smallest splitting step was 0,005 mm, and the largest 2,6 mm ($\Delta x_{\min} = 0,005$ mm; $\Delta y_{\min} = 0,01$ mm (only the first layer); $\Delta x_{\max} = 2,6$ mm; $\Delta y_{\max} = 0,65$ mm).

Figs. 2, 5, 8, 11, 14, 17; 3, 6, 9, 12, 15, 18; 4, 7, 10, 13, 16, 19 show the fields of the Odquist parameter κ, normal stresses σ_{xx} and σ_{yy} at times $t_1 = 2.57 \cdot 10^{-6}$ s, $t_2 = 3.82 \cdot 10^{-6}$ s and $t_3 = 5.13 \cdot 10^{-6}$ s, respectively.

From Figs. 2 – 7 it can be seen that in the area under the contact zone the plastic deformations are bigger and quicker in the case of PStS and at the end of the process of non-stationary interaction, when the moment of time t_3 they are of the higher degree.

Figs. 8 – 19 show that the highest stresses occur in the upper layer of the metal and the process of accumulation of plastic deformations is more intense there. These Figs. show areas where the normal stresses in layers are tensile. This is due to the fact that compressive stresses arise in the upper layer quickly and the contact between the layers and the contact of the lower boundary of the lower layer with an absolutely rigid base are ideally rigid.

The summary plastic deformations at time t_1 in the case of PStS are 30% greater than in the case of PSS and the area where these plastic deformations occur is slightly larger. At times t_2 and t_3, the area of plastic deformations in the case of PSeS is located under the contact zone, and the summary plastic deformations are greater in magnitude than in the case of PSS by 32% and 98% at times t_2 and t_3, respectively. In the case of PStS, the large in absolute value normal stresses σ_{xx} and σ_{yy} arise in the area under the contact zone. Moreover, the values of normal stresses σ_{xx} in the case of PStS are less in absolute value than the values in the case of

Fig. 2. PStS. Odquist parameter κ when $t = t_1$

Fig. 3. PStS. Odquist parameter κ when $t = t_2$

Fig. 4. PStS. Odquist parameter κ when $t = t_3$

Fig. 5. PSS. Odquist parameter κ when $t = t_1$

Fig. 6. PSS. Odquist parameter κ when $t = t_2$

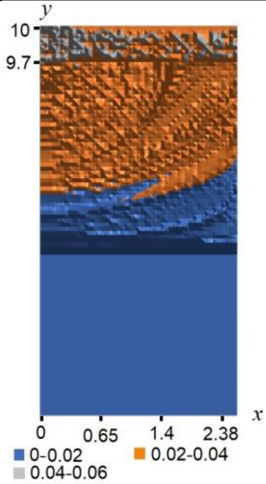

Fig. 7. PSS. Odquist parameter κ when $t = t_3$

Fig. 8. PStS. Stress σ_{xx} when $t = t_1$

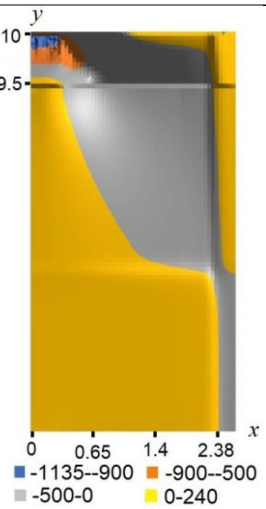

Fig. 9. PStS. Stress σ_{xx} when $t = t_2$

Fig. 10. PStS. Stress σ_{xx} when $t = t_3$

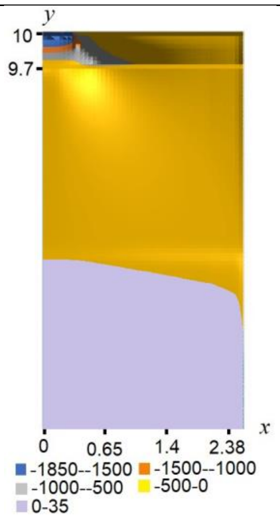

Fig. 11. PSS. Stress σ_{xx} when $t = t_1$

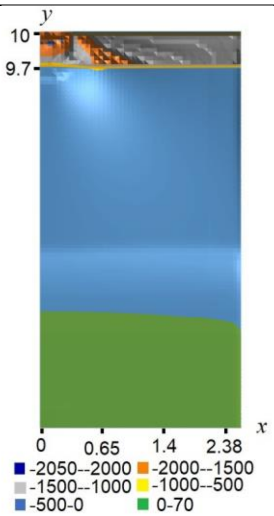

Fig. 12. PSS. Stress σ_{xx} when $t = t_2$

Fig. 13. PSS. Stress σ_{xx} when $t = t_3$

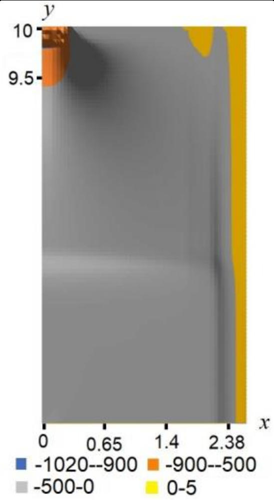

Fig. 14. PStS. Stress σ_{yy} when $t = t_1$

Fig. 15. PStS. Stress σ_{yy} when $t = t_2$

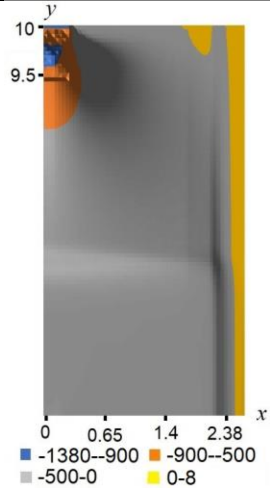

Fig. 16. PStS. Stress σ_{yy} when $t = t_3$

Fig. 17. PSS. Stress σ_{yy} when $t = t_1$

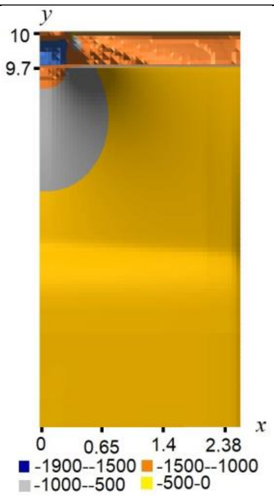

Fig. 18. PSS. Stress σ_{yy} when $t = t_2$

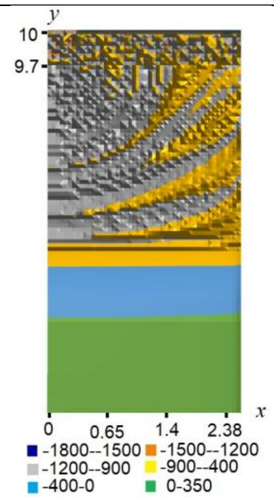

Fig. 19. PSS. Stress σ_{yy} when $t = t_3$

PSS at the times t_1, t_2 and t_3, respectively, by 40%, 45% and 43%. The absolute values of normal stresses σ_{yy} in the case of PStS are less than the corresponding values in the case of PSS at the same time points by 22%, 36% and 24%, respectively.

The PIeS simulates the process of impact on a narrow strip of a two-layer base. In the case of PSeS, plastic deformations grow much faster than in the case of PSS.

CONCLUSIONS

The developed methodology of solving dynamic contact problems in an elastic-plastic dynamic mathematical formulation makes it possible to model the processes of impact, shock and non-stationary contact interaction with the elastic composite base more adequately. In this work, the process of impact on a two-layers base, consisting of an upper thin layer of metal and a lower main layer of glass, is adequately modelled and investigated. The fields of summary plastic deformations and normal stresses arising in the base are calculated and compared to the corresponding values from the corresponding problem of plane strain state. The upper metal layer of the composite two-layer base takes on the main load. The results obtained make it possible to design the narrow strips of new composite reinforced armed materials. Such a two-layer reinforced composite material can be used as a wide range of needs of modern industry.

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Задача о плоском деформированном состоянии двухслойного тела в динамической упругопластической постановке

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Аннотация. Композитные материалы широко используются в промышленности и быту. Для расчета и разработки композиционных материалов используется множество различных методов. Успешно используются разные методы расчета и проектирования таких материалов. В данной статье для проектирования композиционных и армированных материалов используется методика решения динамических контактных задач в более точной упругопластической математической постановке. Для учета физической нелинейности процесса деформирования используется метод последовательных приближений, позволяющий свести нелинейную задачу к решению последовательности линеаризованных задач. В рамках динамической упругопластической математической модели решается задача о плоском напряженном состоянии (ПНС) балки из композиционного армированного двухслойного материала. Армированный или усиленный материал состоит из двух слоев: верхнего (первого) тонкого слоя из твердой стали и нижнего (второго) основного слоя из стекла. Это композиционное основание жестко связано с абсолютно твердым полупространством. Предполагается жесткое сцепление слоев друг к другу. Стекло – очень прочный и в то же время очень хрупкий материал. Хрупкость стекла связана с тем, что на поверхности имеется множество микротрещин, и при приложении нагрузки к поверхности стекла эти микротрещины начинают расти и приводят к разрушению стеклянных изделий. Если склеить или зафиксировать вершины микротрещин на поверхности, то получится прочный армированный усиленный материал, который будет легче, прочнее и не будет подвержен деградации свойств материала, таких как старение, коррозия и ползучесть.

Процесс удара моделировался как нестационарная задача о плоском напряженном состоянии с равномерно распределенной нагрузкой в области контакта, изменяющейся по линейному закону. Поля значений параметра Одквиста и нормальных напряжений изучены и сопоставлены с соответствующими результатами задачи о плоской деформации (ПДС) с тем же материалом слоев, их толщиной и размером контактной площадки. На поверхность стекла можно нанести верхний армирующий слой из металла или стали, чтобы атомы металла или стали проникали глубоко, заполняли микротрещины и связывали их вершины. Верхний слой может быть довольно тонким.

Ключевые слова: Плоская деформация, удар, композитные материалы, армированные материалы, бронированные материалы, упругопластическая, деформация.